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DEVELOPMENT OF COLLABORATIVE MATHEMATICS LEARNING MODEL GOGAR INTEGRATED WITH ETNOMATHEMATICS TO IMPROVE REASONING ABILITY

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ABSTRACT

Tujuan penelitian ini adalah untuk mengetahui: (1) kualitas Model Pembelajaran Matematika Kolaboratif GoGAR terintegrasi Etnomatematika yang dikembangkan, dan (2) peningkatan kemampuan penalaran matematis siswa melalui Model Pembelajaran Matematika Kolaboratif GoGAR terintegrasi Etnomatematika. Penelitian ini merupakan penelitian pengembangan dengan menggunakan model pengembangan Thiagarajan, yaitu model 4D yang dimodifikasi (Define, Design, Develop, dan Disseminate). Selain model pembelajaran, dikembangkan pula perangkat pembelajaran untuk mendukung pelaksanaan model, berupa RPP, LKS, buku siswa, serta instrumen untuk menguji kemampuan penalaran matematis siswa. Instrumen penelitian meliputi: (a) lembar validasi model dan perangkat pembelajaran; (b) lembar observasi untuk melihat kemampuan guru dalam mengelola pembelajaran dan aktivitas siswa; serta (c) angket penelitian untuk mengetahui respons siswa terhadap model pembelajaran yang dikembangkan. Hasil analisis data menunjukkan bahwa Model Pembelajaran Matematika Kolaboratif GoGAR terintegrasi Etnomatematika memenuhi kriteria valid, praktis, dan efektif. Terdapat peningkatan kemampuan penalaran matematis siswa dengan menggunakan model pembelajaran yang dikembangkan.

The purpose of this study was to determine: (1) the quality of the Collaborative Mathematics Learning Model GoGAR integrated with Ethnomathematics that was developed, and (2) to determine the increase in students' mathematical reasoning abilities with the Collaborative Mathematics Learning Model GoGAR integrated with Ethnomathematics. This study is a development research using the Thiagarajan development model, namely the modified 4D model (Define, Design, Develop, and Disseminate). In addition to the model, learning tools were also developed to support the implementation of the model in the form of RPP, LKS, student books, and instruments to test students' mathematical reasoning abilities. The research instruments were in the form of: (a) model validation sheets and learning tools; (b) observation sheets to see the teacher's ability to manage learning and student activities; (c) research questionnaires *to see students' responses to the learning model that was developed. The results of the data analysis showed that the Collaborative Mathematics Learning Model GoGAR integrated with Ethnomathematics met the criteria of valid, practical, and effective. There was an increase in students' mathematical reasoning abilities using the developed learning model.*

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1. INTRODUCTION

Mathematical reasoning is a central objective of contemporary mathematics education and is widely acknowledged as a key component of higher-order thinking skills. Reasoning enables students to construct logical arguments, justify mathematical claims, and evaluate the validity of conclusions. Recent international studies consistently report that students' mathematical reasoning abilities remain underdeveloped, particularly when instruction emphasizes procedural execution rather than conceptual understanding and justification (Isik & Oz, 2024), (Mukuka et al., 2023).

One major cause of weak mathematical reasoning is the persistence of teacher-centered instructional practices that limit students' opportunities to articulate ideas, justify solutions, and engage in mathematical argumentation. Such practices position students as passive recipients of knowledge rather than active reasoners. Empirical evidence indicates that reasoning skills develop more effectively in learning environments that emphasize dialogue, collaboration, and justification of ideas (Mukuka et al., 2023), (Ryan, 2022).

In recent years, ethnomathematics has gained increasing attention as a pedagogical approach that connects mathematical concepts with students' cultural experiences. By situating mathematics within familiar cultural practices, ethnomathematics enhances meaningful learning, student engagement, and conceptual understanding. Numerous studies have demonstrated that ethnomathematics-based learning contributes positively to students' reasoning and problem-solving abilities (Maidiyah et al., 2021), (Jampel & Antara, 2025), (Zainovi et al., 2025).

Despite these positive findings, most ethnomathematics studies focus primarily on contextual relevance and learning outcomes, while providing limited insight into how students construct, justify, and negotiate mathematical reasons during learning activities. Many instructional models integrate culture as context but do not explicitly design learning interactions that require students to engage in structured reasoning and argumentation (Ngala & Marsigit, 2024).

From a theoretical standpoint, inferentialism offers a robust framework for conceptualizing mathematical learning as a process of reasoning through social interaction. Inferentialism views meaning and understanding as emerging from participation in practices of giving, asking for, and evaluating reasons. Within mathematics education, this perspective highlights the importance of argumentation, justification, and dialogical engagement as core mechanisms of learning (Seidouvy & Schindler, 2019), (Drimalla, 2025).

An instructional approach that explicitly operationalizes inferentialism is the Game of Giving and Asking for Reasons (GoGAR)**.** GoGAR emphasizes collective reasoning by encouraging students to present claims, justify their thinking, and critically respond to peers' arguments. Research has shown that GoGAR-based learning environments support the development of inferential reasoning and enhance students' participation in mathematical discourse (Eckert & Nilsson, 2021). However, existing GoGAR studies are largely limited to discourse analysis or technology-enhanced settings and rarely integrate cultural contexts explicitly.

Taken together, the existing literature reveals a clear research gap. While ethnomathematics has been shown to enhance contextual understanding and collaborative learning has been shown to support reasoning, few studies have developed and validated a learning model that systematically integrates ethnomathematics with structured inferential practices such as GoGAR. In particular, there is a lack of development research that embeds GoGAR within culturally grounded mathematics learning to explicitly foster students' mathematical reasoning abilities.

To address this gap, the present study develops a Collaborative Mathematics Learning Model based on GoGAR integrated with Ethnomathematics. The novelty of this study lies in its explicit integration of (1) ethnomathematics as a cultural-contextual foundation and (2) GoGAR as a structured inferential mechanism that requires students to give, ask for, and evaluate mathematical reasons collaboratively. The model is developed and evaluated using a modified 4D Research and Development framework, focusing on its validity, practicality, and effectiveness in improving students' mathematical reasoning abilities.

2. METHOD

2.1. Types Of Research

This study employed a Research and Development (R&D) approach aimed at developing, validating, and evaluating a Collaborative Mathematics Learning Model based on the Game of Giving and Asking for Reasons (GoGAR) integrated with Ethnomathematics. R&D is widely used in mathematics education research to generate theoretically grounded and empirically validated instructional models that are feasible for classroom implementation (Plomp, 2013); (Nieveen & Folmer, 2019).

The development procedure followed a modified 4D model consisting of Define, Design, Develop, and Disseminate stages. The 4D model was selected because it provides a systematic framework for aligning theoretical foundations, instructional design, and empirical evaluation, and has been extensively adopted in recent mathematics education development studies (Sari et al., 2023), (Hidayat et al., 2022).

In this study, the dissemination stage was limited to a restricted trial due to contextual constraints, which is consistent with exploratory R&D research in school-based settings.

The research was conducted at a public junior high school during the 2023/2024 academic year. Participants consisted of 32 eighth-grade students selected through purposive sampling. This sampling strategy was chosen to ensure alignment between the developed model and the students' curriculum level and cultural background relevant to the ethnomathematics content.

Although the study involved a single class, which may limit generalizability, recent development research emphasizes that initial validation of instructional models may legitimately employ small-scale trials, provided that limitations are transparently acknowledged and results are interpreted cautiously (Plomp & Nieveen, 2013). To mitigate this limitation, multiple data sources and triangulation techniques were employed.

The define stage involved analyzing students' learning difficulties, curriculum requirements, and existing instructional practices related to mathematical reasoning. Needs analysis was conducted through classroom observations, interviews with teachers, and document analysis of lesson plans and assessment instruments. This stage ensured that the developed model addressed authentic classroom problems, particularly students' limited engagement in reasoning and argumentation processes (Mukuka et al., 2023).

At this stage, the structure of the GoGAR-based collaborative learning model was designed, including learning syntax, social system, teacher roles, student activities, and supporting learning materials. Ethnomathematical contexts were embedded into learning tasks to promote meaningful learning and cultural relevance, as recommended by recent studies (Ngala & Marsigit, 2024). The design explicitly incorporated inferential practices, requiring students to present claims, provide justifications, and respond to peers' reasoning.

The develop stage focused on expert validation, limited trials, and revision of the learning model. Validation involved three experts in mathematics education who evaluated the model's content validity, construct validity, and instructional feasibility using structured validation instruments. The validity criteria followed the framework proposed by Nieveen, which emphasizes internal consistency and theoretical alignment (Nieveen & Folmer, 2019).

2.2 Data Collector

Expert validation sheets were used to assess the validity of the learning model and instructional materials. Inter-rater agreement was examined to ensure consistency of expert judgments, following recommendations in instructional design research (Akker et al., 2020).

Practicality Instruments

Practicality was measured using:

1. Observation sheets of learning implementation
2. Teacher response questionnaires

3. Student response questionnaires

Practicality assessment focused on ease of use, clarity of instructions, and feasibility of classroom implementation, consistent with recent R&D studies in mathematics education (Hidayat et al., 2022).

Students' mathematical reasoning ability was measured using a reasoning test administered before and after the intervention. The test assessed indicators such as making conjectures, providing justifications, drawing logical conclusions, and evaluating arguments. These indicators align with contemporary conceptualizations of mathematical reasoning (Isik & Oz, 2024).

Content validity was examined through expert review, while reliability was assessed using Cronbach's alpha, consistent with best practices in educational measurement (Tavakol & Dennick, 2011).

2.3 Data Analysis Technique

Validity and practicality data were analyzed descriptively using mean scores and categorical criteria. The effectiveness of the learning model was evaluated using normalized gain (N-gain) analysis to determine the magnitude of improvement in students' mathematical reasoning abilities. N-gain is commonly used in development research to assess learning improvements in small-scale trials (Hake, 1998), (Sari et al., 2023).

Although inferential statistical tests were not applied due to the limited sample size, the results provide preliminary evidence of the model's effectiveness. This approach is consistent with exploratory R&D studies, where the primary objective is to evaluate feasibility and potential impact rather than causal inference.

1. Model Validity Test

The validity of the developed learning model will be assessed by two validators by providing a validity assessment questionnaire. The assessment questionnaire consists of 4 scores, with the following criteria:

Table 1. Score Intervals and Validity Categories of the Learning Model

Score Interval	Assessment Category	Keterangan
$3,50 \leq P \leq 4,00$	Very Valid	Can be used without revision
$2,75 \leq P < 3,50$	Valid	Can be used with revision
$2,00 \leq P < 2,75$	Less Valid	Can be used with major revisions
$1,00 \leq P < 2,00$	Not Valid	Can not be used

The level of data agreement (R) between two observers was carried out using the interobserver agreement method with consideration of the percentage of agreement:

$$R = 100\ (1 - \frac{A-B}{A+B})$$ (Nieeven, N. & Plomp, T. (2007)

With : R = Reliability value
A = Validator assessment that gives high marks
B = Validator assessment that gives low scores

1. Model Practicality Test

The practicality of the model is seen from (a) the implementation of the model during the trial and (b) the active activities of students during the implementation of the learning model.

a. Model Implementation

The implementation of the learning model is seen through the model observation questionnaire. This questionnaire is compiled according to the syntax of the GoGAR collaborative learning model integrated with Ethnomathematics. The model implementation questionnaire will be used by two observers consisting of 4 scores, namely score 4 = good; score 3 = quite good; score 2 = less good, and score 1 = not good.

The results of the observer's observations will be calculated with the following assessment criteria:

Table 2. Criteria for Assessing the Implementation of the Learning Model

Score Interval	Assessment Category
3,50 ≤ P ≤ 4,00	Very Good
2,75 ≤ P < 3,50	Good
2,00 ≤ P < 2,75	Good Enough
1,00 ≤ P < 2,00	Not Good

b. Active Student Activities

Students' active activities during the learning process are seen through activity observation questionnaires. The calculation of the percentage of activity uses the following formula:

$P\ (\%) = \frac{A_0}{A_1}\ x\ 100\%$, With value P at least be at 75%.

3. Model Effectiveness Test

Model effectiveness is seen from: (a) Student response to the learning model and (b) student learning completion using the developed model.

a. Student Response

Student responses are analyzed based on the Guttman scale which is expressed in the form of a student questionnaire. The completed questionnaire is then calculated using the formula:

$$\text{Respon} = \frac{Jumlah\ siswa\ menjawab\ Ya}{Jumlah\ seluruh\ siswa}\ x\ 100\%$$

Table 3. Criteria for Assessing Student Responses to the Learning Model

Percentage Interval	**Assessment Category**
0 % ≤ Response ≤ 20%	Not good
20 % < Response ≤ 40%	Prety good
40 % < Response ≤ 60%	Good Enough
60 % < Response ≤ 80%	Good
80 % < Response ≤ 100%	Very Good

The developed model is said to receive a response if it is at least in the "Good" category.

b. Student Learning Completeness

Student learning completeness is seen from class completeness, namely at least 80% of students achieve KKM. The results of the student's pretest and posttest were analyzed using normalized gains descriptive analysis.

$$N_{gain} = \frac{Postes - Pretes}{Skor\ Max - Pretes}$$

With the criteria:

Table 4. N-Gain Value Criteria

N-gain value	Criteria
N-gain ≥ 0,70	high
0,30 ≤ N-gain < 0,70	Currently
0,00 ≤ N-gain < 0,30	Low

Ethical approval was obtained from the school authority. All participants and their guardians provided informed consent prior to data collection. Confidentiality and anonymity of student data were strictly maintained in accordance with ethical guidelines for educational research (BERA, 2018).

3. RESULT AND DISCUSSION

3.1. Results

1. Model Validity Test

The content validity of the GoGAR Integrated Ethnomathematics collaborative learning model was obtained through the model content validity assessment instrument. In detail, the results of the content validity assessment of the developed learning model can be seen in the following table 4.

Table 5. Model Validity Test

No	Assessment Aspects	Validity Results		Reliabilitas	
		Average	Criteria	R	K
I	**Rationale for GoGAR Collaborative Learning Model Integrated with Ethnomathematics**				
	1. The objective of the GoGar Integrated Ethnomathematics collaborative learning model is in accordance with the needs of 21st century skills graduate competencies.	3,33	V	0,86	R
	2. Development of the GoGar Integrated Ethnomathematics Collaborative Learning Model bridges the gap between expectations and the competency needs of 21st century graduates.	3,33	V	0,86	R
	3. The GoGar Integrated Ethnomathematics Collaborative Learning Model meets the needs of the learning process which prioritizes activities of giving and asking for reasons in drawing conclusions.	3,67	SV	0,86	R
	4. Development of the GoGar Integrated Ethnomathematics collaborative learning model taking into account recommendations for improvement from research.	4	SV	1,00	R
	5. The GoGar Integrated Ethnomathematics Collaborative Learning Model uses a transdisciplinary approach to meet state of the art scientific knowledge.	3,67	SV	0,86	R
II	**Theoretical and empirical support**				
	1. The novelty of the GoGar Integrated Ethnomathematics collaborative learning model was built by considering the advantages and limitations of other models.	3,33	V	0,86	R
	2. Development of the objectives of the GoGar Integrated Ethnomathematics collaborative learning model using primary sources from reputable journals.	3,67	SV	0,86	R
	3. Development of the GoGar Integrated Ethnomathematics collaborative learning model using theoretical foundations from standard and current educational psychology figures.	3,67	SV	0,86	R
	4. Development of the GoGar Integrated Ethnomathematics collaborative learning model using empirical foundations obtained from studies and references from various relevant research.	3,67	SV	0,86	R
III	**Model Syntax**				
	1. The GoGar Integrated Ethnomathematics learning model was developed with the main objective of facilitating students' mathematical reasoning abilities.	4,00	SV	1,00	R
	2. The syntax of the GoGar Integrated Ethnomathematics collaborative learning model can be categorized as a new syntax.	3,33	V	0,86	R
	3. The GoGar Integrated Ethnomathematics collaborative learning model learning environment supports the achievement of optimal learning.	3,33	V	0,86	R

No	Assessment Aspects	Validity Results		Reliabilitas	
		Average	Criteria	R	K
4.	Development of assessment and evaluation of the GoGar Integrated Ethnomathematics collaborative learning model using the latest reference sources.	3,33	V	0,86	R

2. Model Practicality Test

Model practicality is seen from the implementation of the GoGar Integrated Ethnomathematics Learning Model that was developed. The results of the model implementation test can be seen in the following figure.

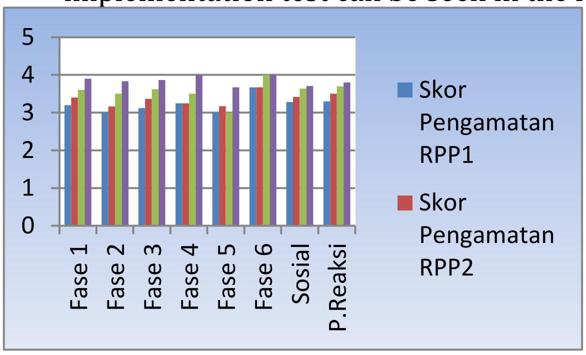

Figure 1. Implementation of Limited Trial Model Trial

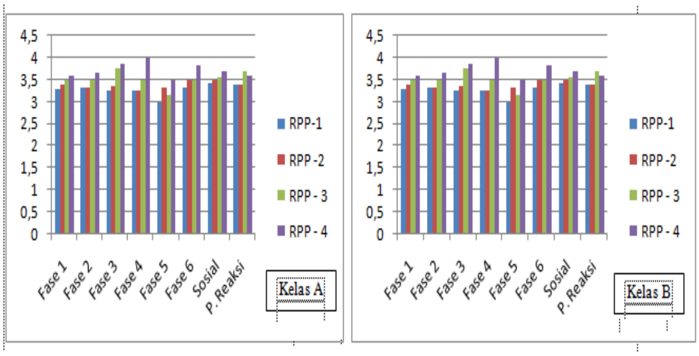

Figure 2. Model Implementation in Wide

From Figure 1 and Figure 2 it can be seen that the implementation of the GoGar Integrated Ethnomathematics learning model for model syntax, social systems and aspects of teacher reaction/behavior principles tend to increase at the last meeting. Based on the figure, it can be concluded that the GoGar Integrated Ethnomathematics collaborative learning model can be implemented well.

3. Model Effectiveness Test

The results of the effectiveness of the GoGAR collaborative learning model integrated with Ethnomathematics can be seen from (a) the results of the student response questionnaire analysis and (b) the results of the student's mathematical reasoning ability test.

a. Student Response Questionnaire

Student responses to learning by implementing the GoGAR Integrated Ethnomathematics learning model are presented in Table 2 below.

Table 6. Student Response Questionnaire Data Regarding Learning in Limited Trial

No	Question Description	Opinion Assessment (%)			
A	Opinions on the GoGar Integrated Ethnomathematics Collaborative Learning Model	Very interesting	Enough	Less	Not interesting
		49.99	43.09	6.89	0.00
B	Do you feel that the components of the lesson material/content, BAS, LKPD and learning atmosphere are new?	Very New	Enough	Less	Not New
		55.16	41.37	3.44	0.00
C		Very Interested	Enough	Less	Not Interested

	Learning method that applies the GoGar collaborative learning model Integrated Ethnomathematics	43.67	45.97	10.34	0.00
D	Do student textbooks help you develop your mathematical skills?	Very helpful	Enough	Less	Not helpful
		52.29	43.67	4.01	0.00
E	Teacher teaching model that applies GoGar Integrated Ethnomathematics collaborative learning model	Very clear	Enough	Less	Not clear
		46.54	48.27	5.16	0.00
F	Your assessment of students' mathematical reasoning abilities	Not Difficult	Enough	Less	Difficult
		21.83	71.26	6.89	0.00
G	Are you having difficulty answering questions or tests in your student textbook?	Not Difficult	Enough	Less	Difficult
		27.58	68.96	3.44	0.00

Next, the results of the student response questionnaire in the extensive trial are presented in Table 3.

Table 7. Results of the Student Response Questionnaire to Learning in the Extensive Trial

No	Uraian Pertanyaan	Penilaian Pendapat (%)			
A	Opinions on the GoGar Integrated Ethnomathematics Collaborative Learning Model	Very interesting	Enough	Less	Not interesting
		45,07	**42,21**	**7,78**	**0,81**
B	Do you feel that the components of the lesson material/content, BAS, LKPD and learning atmosphere are new?	Very New	Enough	Less	Not New
		50,81	**43,84**	**5,32**	**0,00**
C	Learning method that applies the GoGar collaborative learning model Integrated Ethnomathematics	Very Interested	Enough	Less	Not Interested
		39,88	**50,26**	**12,56**	**2,72**
D	Do student textbooks help you develop your mathematical skills?	Very helpful	Enough	Less	Not helpful
		53,82	**40,15**	**5,73**	**0.27**
E	Teacher teaching model that applies GoGar Integrated Ethnomathematics collaborative learning model	Very clear	Enough	Less	Not clear
		45,08	**43,44**	**10,65**	**0.81**
F	Your assessment of students' mathematical reasoning abilities	Not Difficult	Enough	Less	Difficult
		21,30	**64,47**	**13,65**	**0,54**
G	Are you having difficulty answering questions or tests in your student textbook?	Not Difficult	Enough	Less	Difficult
		21,31	**60,65**	**14,75**	**3,27**

b. Students' Mathematical Reasoning Ability Test

The results of students' mathematical reasoning ability test using the GOGAR collaborative learning model integrated with Ethnomathematics can be seen in the following table.

Table 8. Students' Mathematical Reasoning Test Results Based on N-Gain Analysis

Trials	Description	Value	N-Gain
Limited	Pre Test	356	**0,57**
	Post Test	976	
	Max Score	1440	
Comprehensive	Pre Test	413	**0,66**
	Post Test	1042	

Max Score	1360

By looking at the table above, it can be concluded that there is an increase in students' mathematical reasoning abilities by using the GoGAR collaborative learning model integrated with Ethnomathematics with a "moderate" increase category in limited trials and extensive trials

3.2. Discusion

The findings of this study indicate that the Collaborative Mathematics Learning Model based on GoGAR integrated with Ethnomathematics meets the criteria of validity, practicality, and effectiveness, and demonstrates a positive impact on students' mathematical reasoning abilities. These results suggest that integrating structured inferential practices within culturally contextualized collaborative learning can meaningfully enhance students' engagement in mathematical reasoning processes.

The improvement in students' mathematical reasoning can be explained through the lens of inferentialism, which conceptualizes learning as participation in social practices of giving, asking for, and evaluating reasons. By engaging students in GoGAR-based activities, the learning model explicitly required them to articulate claims, justify solutions, and respond to peers' arguments. This aligns with recent studies showing that reasoning develops more effectively when students are actively involved in argumentation and justification rather than passively receiving information (Drimalla, 2025), (Isik & Oz, 2024).

Furthermore, the collaborative structure of the GoGAR model facilitated dialogical interactions that supported collective reasoning. Prior research emphasizes that collaborative learning environments foster reasoning by enabling students to negotiate meanings, challenge assumptions, and co-construct mathematical understanding (Mukuka et al., 2023), (Ryan, 2022). The present findings reinforce these conclusions by demonstrating that collaboration, when guided by explicit inferential rules such as those embedded in GoGAR, leads to more purposeful and reasoning-oriented discourse.

The integration of ethnomathematics further strengthened the effectiveness of the learning model by situating mathematical reasoning within culturally familiar contexts. Ethnomathematical tasks enabled students to draw upon prior cultural knowledge, which supported sense-making and reduced cognitive barriers to reasoning. This finding is consistent with recent empirical studies reporting that culturally contextualized mathematics instruction enhances conceptual understanding and reasoning abilities (Maidiyah et al., 2021), (Ngala & Marsigit, 2024), (Jampel & Antara, 2025).

Notably, the moderate N-gain obtained in this study suggests that while the model effectively improved students' reasoning, there remains room for further optimization. Similar studies in mathematics education report moderate gains when introducing reasoning-oriented instructional innovations in initial implementations, particularly in contexts where students are not accustomed to argumentation-based learning (Sari et al., 2023), (Supriadi et al., 2024). This indicates that sustained exposure and iterative refinement of the learning model may yield stronger gains over time.

Compared with previous ethnomathematics-based learning models, the present study contributes a distinctive advancement by explicitly embedding inferential reasoning mechanisms through GoGAR. While earlier studies have demonstrated the benefits of ethnomathematics for engagement and contextual understanding, they often lacked a structured framework for guiding students' reasoning and argumentation (Zainovi et al., 2025). The GoGAR–Ethnomathematics integration addresses this gap by ensuring that cultural context is not merely illustrative, but functions as a catalyst for inferential reasoning.

Despite these promising results, several limitations should be acknowledged. The study was conducted with a limited sample size and without a control group, which constrains the generalizability of the findings. However, this limitation is common in early-stage educational design research, where the primary objective is to establish feasibility and preliminary effectiveness (Plomp & Nieveen, 2013). Future research should involve larger samples, experimental or quasi-experimental designs, and longitudinal implementations to further validate the model's impact.

Overall, the findings of this study support the growing body of evidence that reasoning-oriented, collaborative, and culturally grounded instructional models are effective in enhancing students' mathematical reasoning. By integrating inferentialism, GoGAR, and ethnomathematics, this study contributes a theoretically informed and practically viable model that addresses key challenges in contemporary mathematics education.

4. CONCLUSION

This study aimed to develop and evaluate a Collaborative Mathematics Learning Model based on the Game of Giving and Asking for Reasons (GoGAR) integrated with Ethnomathematics to enhance students' mathematical reasoning abilities. Based on the results of the development and limited implementation, it can be concluded that the proposed model meets the criteria of validity, practicality, and effectiveness, indicating that it is feasible for use in junior secondary mathematics classrooms.

The findings demonstrate that integrating structured inferential practices through GoGAR within culturally contextualized learning environments can positively support students' mathematical reasoning. By requiring students to articulate claims, justify solutions, and critically evaluate peers' arguments, the model aligns with inferentialist perspectives that view reasoning as a socially mediated practice (Drimalla, 2025). The observed improvement in students' reasoning abilities reinforces previous research highlighting the effectiveness of argumentation-based and collaborative learning approaches in mathematics education (Isik & Oz, 2024), (Mukuka et al., 2023).

Moreover, the integration of ethnomathematics played a significant role in enhancing the meaningfulness of learning by connecting mathematical concepts to students' cultural experiences. This cultural grounding facilitated sense-making and engagement, which are essential conditions for the development of higher-order reasoning skills (Maidiyah et al., 2021), (Ngala & Marsigit, 2024). Unlike prior ethnomathematics-based studies that primarily emphasized contextual relevance, this study contributes a novel instructional model that explicitly embeds inferential reasoning mechanisms to guide students' mathematical discourse.

From a theoretical perspective, this study contributes to mathematics education research by demonstrating how inferentialism can be operationalized pedagogically through GoGAR within a culturally responsive learning model. Practically, the developed model offers teachers a structured framework for fostering collaborative reasoning and argumentation in mathematics classrooms, particularly in culturally diverse contexts.

Nevertheless, this study has limitations. The research was conducted on a limited scale with a single class and without a control group, which restricts the generalizability of the findings. Future research should involve larger and more diverse samples, employ experimental or quasi-experimental designs, and examine the long-term effects of the GoGAR–Ethnomathematics model on students' reasoning development. Further studies may also explore the integration of digital technologies to support GoGAR-based inferential practices, as suggested by recent research on technology-enhanced mathematical discourse (Eckert & Nilsson, 2021).

In conclusion, the GoGAR-based collaborative mathematics learning model integrated with ethnomathematics represents a promising instructional innovation that bridges cultural relevance and structured reasoning practices. The model has the potential to contribute meaningfully to the advancement of reasoning-oriented mathematics education and to inform future research and practice in this field.

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