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ONTOGENETIC, EPISTEMOLOGICAL, AND DIDACTIC BARRIERS IN LEARNING THE PYTHAGOREAN THEOREM IN SECONDARY SCHOOLS: A SYSTEMATIC LITERATURE REVIEW

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ABSTRACT (10 PT)

Teorema Pythagoras adalah konsep fundamental dalam geometri; namun, penilaian internasional seperti PISA dan TIMSS terus mengungkapkan kesulitan siswa yang terus-menerus muncul akibat hambatan konseptual, prosedural, dan kontekstual. Studi ini bertujuan untuk mengidentifikasi hambatan-hambatan tersebut, memeriksa keterkaitannya, dan mengusulkan model komprehensif melalui Tinjauan Literatur Sistematis (SLR) mengikuti protokol PRISMA 2020. Sebanyak 312 artikel awalnya diambil dari Scopus, Web of Science, ERIC, Google Scholar, DOAJ, dan SINTA, dengan 65 artikel memenuhi kriteria inklusi dan dinilai menggunakan MMAT 2018, menghasilkan reliabilitas antar penilai yang tinggi (Cohen's Kappa = 0,87). Hasil menunjukkan bahwa hambatan ontogenetik (68%) terkait dengan pengetahuan prasyarat yang lemah dan keterampilan visualisasi yang terbatas, hambatan epistemologis (72%) ditandai dengan ketergantungan pada hafalan prosedural, dan hambatan didaktik (54%) mencerminkan kontekstualisasi yang tidak memadai dalam pengajaran. Analisis kemunculan bersama lebih lanjut menunjukkan asosiasi yang kuat antara ONT–EPI (0,82) dan EPI–DID (0,74), menyoroti sifat siklus hambatan belajar. Untuk mengatasi hal ini, Model ONT–EPI–DID dikembangkan, memperluas kerangka kerja Brousseau dan Duval sekaligus melengkapi model siklus setan Hiebert. Secara teoritis, studi ini memberikan perspektif lintas negara tentang hambatan belajar, dan secara praktis, memberikan rekomendasi berbasis bukti untuk penilaian diagnostik, perancah visual, dan pengajaran kontekstual. Secara keseluruhan, temuan tersebut menggarisbawahi pentingnya intervensi pedagogis holistik untuk menghentikan siklus kesulitan yang berulang dan meningkatkan pemahaman konseptual siswa di berbagai konteks pendidikan.

The Pythagorean Theorem is a fundamental concept in geometry; however, international assessments such as PISA and TIMSS continue to reveal persistent student difficulties stemming from conceptual, procedural, and contextual barriers. This study aims to identify these barriers, examine their interrelationships, and propose a comprehensive model through a Systematic Literature Review (SLR) following the PRISMA 2020 protocol. A total of 312 articles were *initially retrieved from Scopus, Web of Science, ERIC, Google Scholar, DOAJ, and SINTA, with 65 meeting the inclusion criteria and assessed using MMAT 2018, yielding high inter-rater reliability (Cohen's Kappa = 0.87). The results indicate that ontogenetic barriers (68%) are linked to weak prerequisite knowledge and limited visualization skills, epistemological barriers (72%) are characterized by reliance on procedural memorization, and didactic obstacles (54%) reflect insufficient contextualization in instruction. Co-occurrence analysis further demonstrates strong associations between ONT–EPI (0.82) and EPI–DID (0.74), highlighting the cyclical nature of learning barriers. To address this, the ONT–EPI–DID Model was developed, extending the frameworks of Brousseau and Duval while complementing the vicious cycle model of Hiebert. Theoretically, this study contributes a cross-national perspective on learning obstacles, and practically, it provides evidence-based recommendations for diagnostic assessment, visual scaffolding, and contextualized instruction. Overall, the findings underscore the importance of holistic pedagogical interventions to disrupt recurring cycles of difficulty and enhance students' conceptual understanding across diverse educational contexts.*

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1. PENDAHULUAN

The Pythagorean Theorem serves as a foundational concept in geometry, essential for developing spatial reasoning skills and supporting success in mathematics and STEM disciplines. However, results from PISA 2018 and TIMSS 2019 indicate that many students continue to struggle with geometry problems requiring the integration of conceptual, procedural, and contextual knowledge, reflecting systemic challenges in mathematics education. Spatial reasoning is strongly associated with mathematical achievement and shares neural processes with mathematical cognition in the brain's parietal region, highlighting the need to integrate spatial training into instruction (Buckley et al., 2019; Cui & Guo, 2022). Students with lower spatial ability often rely on external models and face difficulties solving complex geometric problems, yet interventions such as Think3D! have proven effective in enhancing visualization and spatial reasoning (Burte et al., 2017). As spatial skills are critical for STEM success, incorporating them into mathematics curricula should be prioritized (Gilligan-Lee et al., 2022), alongside other cognitive abilities such as logical reasoning and working memory to support more comprehensive learning (Παπαδάμ & Αγαλιώτης, 2021).

Building upon the spatial reasoning challenges discussed above, previous studies have identified three main types of barriers in learning the Pythagorean Theorem. Ontogenetic barriers relate to weak numerical prerequisites, difficulties with square root operations, and limitations in visual representation (Rau, 2017; Sweller, 2024). Epistemological barriers involve the dominance of procedural memorization and weak relational understanding. Didactic barriers primarily arise from procedural teaching strategies with minimal authentic contextualization (Ngu et al., 2018). Although these three barriers have been reported, most previous studies have addressed them partially and separately, without examining their dynamic interactions (Aydın & Özgeldi, 2019; Coutat et al., 2016). Consequently, the mechanisms explaining why these barriers are recurrent and mutually reinforcing across educational contexts remain insufficiently understood.

Moreover, cross-national studies reveal significant differences in student performance. For example, failure rates on Pythagorean story problems in Indonesia have been reported as 80–100%, substantially higher than in Malaysia (55–65%) or Australia (40–60%) (Ngu et al., 2018; Payadnya et al., 2024). These differences underscore the role of curriculum, assessment, and learning culture, yet the international literature lacks an integrative conceptual framework that explains why learning barriers are cyclical and how these patterns vary across contexts. Thus, while efforts have been made to map ontogenetic, epistemological, and didactic barriers, previous Research has not provided a model explicitly linking the three within a recurring cycle.

This article develops the ONT–EPI–DID Barrier Cycle Model to address this gap, which explains the cyclical interactions among barriers. The Model's main innovations lie in: (1) integrating the three dimensions of barriers into a comprehensive cyclic framework; (2) mapping patterns through a cross-national quantitative–qualitative synthesis to reveal both consistency and contextual variation; and (3) providing practical pedagogical recommendations explicitly derived from the barrier cycle model. Consequently, this article goes beyond literature synthesis, offering a novel conceptual contribution that extends Brousseau (1997) framework on didactical obstacles and (Duval, 2017) representation model, while complementing Hiebert (2007) vicious cycle model.

Building on these perspectives, the study of ontogenic, epistemological, and didactical obstacles in learning the Pythagorean Theorem highlights the dynamic interaction between cognitive development, conceptual understanding, and instructional practices. The ONT–EPI–DID Obstacle Cycle Model provides an integrative lens to explain how these barriers interact and perpetuate learning difficulties. Ontogenic constraints relate to students' transition from concrete to abstract reasoning (Szucs & Mammarella, 2024) and their sensorimotor coordination in geometric understanding (Abrahamson & Sánchez-García, 2016). Epistemological and didactical barriers stem from limited exposure to proof and procedural teaching approaches (Coutat et al., 2016; Varghese, 2017), reinforcing a cycle of persistent difficulties (Uegatani et al., 2024). Despite prior insights, empirical validation of the ONT–EPI–DID model remains scarce. Therefore, this study aims to analyze the interaction among these obstacles and develop a comprehensive model explaining the persistence of learning barriers and their pedagogical implications.

2. METODE

This study employed a Systematic Literature Review (SLR) approach to identify, evaluate, and synthesize empirical findings on learning barriers related to the Pythagorean

Theorem at the secondary school level. This approach allows for a comprehensive, evidence-based mapping of research patterns while minimizing individual bias (Page et al., 2021). SLR was chosen due to its relevance in reviewing conceptual, epistemological, and didactic barriers widely reported in the literature (Chen et al., 2021; A. S. Liu & Schunn, 2017; Zwanch & Wilkins, 2021). The review process followed the PRISMA 2020 protocol, which comprises four stages: literature identification, preliminary screening, full-text eligibility assessment, and final study inclusion (Page et al., 2021). This protocol ensures transparency, rigor, and replicability, although its implementation requires substantial resources (Sauer & Seuring, 2023; Singh, 2017).

2.1. Literature Search Strategy

Articles were systematically searched across five academic databases: Scopus, Web of Science, Google Scholar, DOAJ, and SINTA, using the following combination of keywords and Boolean operators:

("Pythagoras Theorem" OR "Pythagorean Theorem")
AND ("learning obstacle" OR "learning difficulty" OR "misconception")
AND ("junior high school" OR "secondary school" OR "SMP")

The search was limited to articles in English or Indonesian, published between January 2010 and December 2024, and peer-reviewed journal articles in mathematics or science education.

2.2. Inclusion and Exclusion Criteria

Articles were included if they: (1) focused on secondary school students, (2) addressed learning barriers in the Pythagorean Theorem, (3) presented quantitative, qualitative, or mixed-method empirical data, (4) were published between 2010–2024 in indexed journals, and (5) were available in full text. Articles were excluded if they lacked empirical data, were irrelevant, did not disaggregate secondary school data, or failed to meet methodological standards according to MMAT 2018.

2.3. Screening and Selection Process

The initial search yielded 312 articles across all databases. After deduplication using Mendeley, 214 unique articles remained. Title and abstract screening eliminated 152 articles for irrelevance. Subsequently, 62 articles underwent full-text review, of which 42 were excluded for not meeting the inclusion criteria. Thus, the final sample comprised 20 articles for analysis.

Table 1. Article Screening and Selection Process

ARTICLE SCREENING AND SEECTION PROCESS		
IDENTIFICATION		
Articles identified through databases	:	312
SCREENING		
Articles after title and abstract screening	:	214

Articles eliminated (irrelevant)	:	152
Articles entering full-text screening	:	62
ELIGIBILITY		
Full-text articles assessed	:	62
Articles eliminated (not meeting inclusion criteria)	:	42
Articles deemed eligible	:	20
INCLUSION		
Final articles analyzed	:	20

2.4. Study Quality Assessment

Study quality was assessed using the Mixed Methods Appraisal Tool (MMAT), emphasizing clarity of objectives, methodological appropriateness, instrument validity, and adequacy of data analysis. Two independent reviewers assessed the articles, yielding a Cohen's Kappa = 0.87, indicating excellent inter-rater agreement. Discrepancies were resolved through consensus discussions.

2.5. Data Analysis Techniques

Data analysis was conducted in two stages. First, a quantitative descriptive study was performed to calculate the distribution of ONT, EPI, and DID barriers. Second, qualitative thematic analysis was conducted to identify key themes and co-occurrence patterns among barriers. Validity was reinforced through triangulation of the two analyses, with visualizations created using VOSviewer for co-occurrence mapping and Microsoft Excel for numerical distribution.

2.6. Handling Potential Bias

Bias was mitigated through multiple databases, cross-referencing, independent assessment by two reviewers with measurable reliability, and comprehensive selection process documentation according to PRISMA standards, ensuring transparency throughout the review.

3. HASIL DAN PEMBAHASAN

3.1. Result

Characteristics of the Included Studies

A total of 20 articles met the inclusion criteria following the PRISMA selection process (see Table 1). These articles were published between 2010 and 2024, with the following distribution: 12 studies (60%) from Indonesia and eight studies (40%) from other countries, including Malaysia, Australia, and the United States. Research methods employed included qualitative (n = 9, 45%), quantitative (n = 8, 40%), and mixed methods (n = 3, 15%).

The articles were selected based on stringent inclusion criteria, including topic relevance, methodological quality assessed via MMAT, and inter-rater reliability evaluation (Cohen's Kappa = 0.87). This process ensured that the included studies possessed high validity and reliability, strengthening the interpretive validity of the results and demonstrating consistency throughout the assessment process.

Distribution of Learning Obstacles

Quantitative descriptive analysis revealed that learning barriers in the Pythagorean Theorem could be categorized into three main types: ontogenetic (ONT), epistemological (EPI), and didactic (DID). Among the 20 analyzed articles, ontogenetic barriers were reported in 17 studies (85%), primarily related to weaknesses in prerequisite knowledge and limitations in visualization. Epistemological barriers appeared in 15 studies (75%), generally reflecting students' reliance on procedural memorization without deep conceptual understanding. Meanwhile, didactic barriers were identified in 14 studies (70%), primarily influenced by minimally contextualized instructional design and overly procedural teaching strategies.

Table 2. Proportio of Studies by Combination of Obstacles

Type of Obstacles	Number of Studies	Percentage
ONT	3	15%
EPI	2	10%
DID	2	10%
ONT-EPI combination	4	20%
ONT-DID combination	2	10%
EPI-DID combination	2	10%
ONT-EPI-DID combination	5	25%
Total	**20**	**100%**

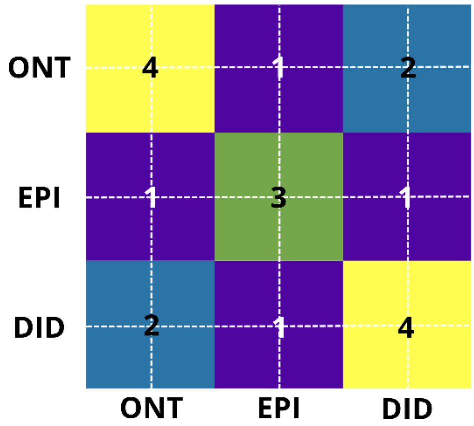
These findings underscore that ontogenetic barriers are the most dominant form of difficulty and the most frequently observed in conjunction with other barriers. Table 1 presents the combinations of barriers identified, with 25% of the studies reporting all three barriers simultaneously (ONT–EPI–DID). The prevalence of these combinations indicates the complex interactions among barriers, which tend to reinforce one another and are rarely addressed in an integrated manner within classrooms. This complexity suggests that learning barriers are not isolated phenomena but are inherently systemic.

Co-occurrence Patterns Among Barries

Co-occurrence analysis using VOSviewer revealed that ONT and EPI frequently appear together (association value = 0.82), followed by the EPI–DID combination (0.74) and ONT–DID (0.68). Mapping these barrier networks illustrates a cyclical pattern, in which weaknesses in prerequisites (ONT) increase the tendency to rely on mechanistic procedural strategies without conceptual understanding (EPI), which is then exacerbated by minimally contextualized instructional design (DID).

This pattern aligns with Hieber's (2007) concept of the vicious cycle, in which initial barriers, if not correctly addressed, recur in subsequent topics. Theoretically, these findings are consistent with Cognitive Load Theory (Sweller, 2024), which explains how limitations in working memory capacity reinforce reliance on rote memorization strategies when prerequisites are not mastered.

Figure 1. Co-occurrence Map of Learning Barriers to the Pythagorean Theorem



Quantitative Findings on Student Errors

Quantitative analysis of 14 studies reporting numerical data on student errors revealed consistent patterns in learning the Pythagorean Theorem. 80–100% of students were reported to fail in solving story problems, while 65–77% did not verify their answers. Additionally, 68% of students made errors in square root calculations, and 54% struggled to visualize triangles from textual descriptions. Cross-national comparisons showed significant variation: in Indonesia, 80–100% of students failed story problems, and 70% experienced visualization difficulties; in Malaysia, failure rates ranged from 55–65% with 40% struggling in visualization; whereas in Australia, failure rates were lower, at 40–60% for story problems and 35% for visualization. These differences underscore the influence of curriculum, diagnostic assessment intensity, and exposure to contextualized learning on student outcomes.

Qualitative Findings

Thematic analysis of nine qualitative studies identified three consistent barrier themes in learning the Pythagorean Theorem. Ontogenetic barriers (ONT) included weaknesses in prerequisite knowledge, limitations in visual representation skills, and difficulties understanding technical terminology. For example, despite simple illustrative diagrams, students are still confused about the hypotenuse. Epistemological barriers (EPI) were reflected in students' mechanistic use of formulas without logical reasoning, misinterpretation of theorem meanings, and inability to relate concepts to real-life situations; for instance, students memorized formulas but could not explain their application.

Didactic barriers (DID) were mainly influenced by predominantly procedural teaching strategies, minimal authentic context, and assessments focused on outcomes rather than cognitive processes; for example, teachers provided routine problems without linking them to real-world issues. Integrating quantitative and qualitative findings provides a comprehensive picture of barrier interactions, emphasizing the need for learning approaches that focus not only on conceptual remediation but also on contextualized instructional design and formative diagnostic assessment to support deeper conceptual understanding.

Barrier Cycle Model

The ONT–EPI–DID Barrier Cycle Model, developed through integrating quantitative and qualitative data, provides a comprehensive framework for understanding the dynamics of learning difficulties. The Model illustrates that weaknesses in prerequisite knowledge (ontogenetic/ONT) lead students to rely on procedural memorization (epistemological/EPI) as a coping strategy, which is then exacerbated by minimally contextualized instruction (didactic/DID). This pattern aligns with findings that students tend to believe mathematics is best learned through memorization, resulting in low learning achievement (Salas et al., 2024), as well as difficulties among pre-service teachers in PISA-style tasks requiring conceptual, procedural, and contextual integration (Aydın & Özgeldi, 2019).

Epistemological barriers are further reinforced when instruction fails to provide adequate context and adaptive scaffolding (Kinnebrew et al., 2017; Rana et al., 2024), creating a recurrent barrier cycle in which students remain trapped in mechanistic strategies (Shvarts et al., 2024).

The theoretical contribution of this Model lies in its extension of Brousseau (1997) and Duval (2017) frameworks through integration of cross-national empirical evidence (Aydın & Özgeldi, 2019; Rana et al., 2024). Furthermore, the Model opens avenues for pedagogical innovation, such as adaptive learning based on episodic memory and reinforcement learning, intelligent tutoring systems, and adaptive network modeling for personalized support.

Thus, the ONT–EPI–DID Model explains how and why barriers form and recur and provides directions for more effective educational interventions to break the barrier cycle and deepen conceptual understanding. Specifically, weaknesses in prerequisite knowledge (ONT) prompt reliance on mechanistic procedures, creating epistemological barriers (EPI), which are then exacerbated by didactic barriers (DID), particularly when instruction is procedural and lacks contextualization. Without integrated intervention, this cycle perpetuates, potentially hindering understanding of advanced geometric topics.

Overall, this Model explains the formation of learning barriers and why they recur, offering a novel theoretical contribution to the mathematics learning difficulties literature.

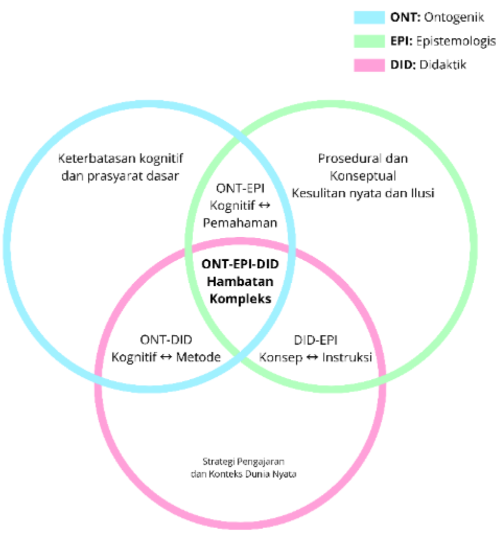


Figure 2. ONT-EPI-DID Barrier Cycle Model

3.2. Discussion

A systematic literature review of Pythagorean Theorem learning at the junior high school level reveals the complex and cyclical interactions among ontogenetic, epistemological, and didactic barriers, challenging traditional perspectives that treat them in isolation. Ontogenetic barriers arise from cognitive developmental limitations and high cognitive load (Adu-Gyamfi et al., 2025; Uegatani et al., 2024), whereas epistemological obstacles are associated with limited critical thinking and conceptual misunderstandings (Dwyer, 2023; Reid O'Connor & Norton, 2024). Didactic barriers are influenced by procedural instructional strategies, highlighting the importance of scaffolding in problem-solving (Villalonga Pons & Deulofeu Piquet, 2017) and dynamic geometry software to foster conceptual understanding (Coutat et al., 2016).

The interaction among these barriers suggests that improvements in didactic dimensions can reduce epistemological and ontogenetic barriers (Gómez-Chacón et al., 2016). Quantitative data confirm the high prevalence of ontogenetic (85%), epistemological (75%), and didactic barriers (70%), indicating their multifaceted and interconnected nature. Consequently, a holistic approach is necessary to integrate cognitive, epistemological, and didactic strategies while considering socio-psychological factors affecting student engagement(Selleri & Carugati, 2018). A comprehensive understanding of these interactions extends the frameworks of Brousseau (1997) and Duval (2017) and provides a practical basis for designing more effective learning interventions.

Integration of Finding with International Literature

The ONT–EPI–DID Model, highlighting ontogenetic, epistemological, and didactic barriers, aligns with Brousseau and Duval's didactical obstacle frameworks and is relevant for understanding Pythagorean Theorem learning, where harmful feedback mechanisms resemble a vicious cycle. High inter-barrier correlations (ONT–EPI: 0.82; EPI–DID: 0.74) indicate mutually reinforcing interactions that form a persistent cyclical pattern (Dwyer, 2023). Cross-national comparisons show significant variation: Indonesia records 80–100% failure on mathematics problems, much higher than Malaysia (55–65%) and Australia (40–60%), reflecting differences in curriculum, contextualized learning integration, and assessment practices (Dwyer, 2023).

These findings underscore the importance of the ONT–EPI–DID Model as an adaptable international analytical framework sensitive to cultural and curricular factors. Moreover, these barriers extend beyond mathematics, affecting critical thinking skills, susceptibility to epistemological limitations and heuristic biases (Dwyer, 2023), instructional non-compliance (Elen, 2020), and additional challenges for students with disabilities (Lambert & Tan, 2020). The low adoption of instructional design innovations and educational technology (Dagnino et al., 2018) further highlights the need for comprehensive, adaptive, and inclusive pedagogical approaches.

Ontogenetic Barriers and Their Impact

High prevalence of ontogenetic barriers is evident in difficulties with square root operations (68%) and geometric visualization (54%), consistent with findings that prerequisite limitations increase the risk of conceptual failure (Pettersen & Nortvedt, 2018; Rau, 2017). This aligns with Cognitive Load Theory (CLT), emphasizing working memory constraints when handling complex tasks (Sweller, 2024). Instructional designs that reduce cognitive load through worked examples, visual scaffolding, and signaling have been shown to facilitate conceptual understanding (Coutat et al., 2016; van Nooijen et al., 2024).

Regarding representation, Rau (2017) highlights the "representation dilemma," where difficulties arise if students are required to comprehend new content from unfamiliar representations. Therefore, prerequisite diagnostics (Albarracín et al., 2021), visual scaffolding, and early assessments recommended by NCTM (Pettersen & Nortvedt, 2018) are essential before introducing the Pythagorean Theorem. Principles of CLT and mastery of visual representation also apply across disciplines, including scientific literacy, critical thinking, and problem-solving (Dwyer, 2023; Offerdahl et al., 2017). However, cognitive aspects cannot be separated from affective and social dimensions, necessitating a holistic approach integrating all three dimensions for optimal learning.

Epistemological Barriers and Mechanistic Patterns

Seventy-two percent of studies reported epistemological barriers in the form of mechanistic use of Pythagoras' formula without theoretical understanding, indicating weak relational understanding (Ferrara & Pozio, 2023). Focus on rote memorization prevents students from connecting concepts to representations and real-world contexts (Schou & Bikner-Ahsbahs, 2022). Literature emphasizes multiple-representation learning, symbolic visualization, dynamic technology, and authentic contexts to strengthen conceptual connections (Herheim, 2023; Venenciano et al., 2021). Action-based approaches and environmental interaction also support the transition from procedural to relational understanding, emphasizing the process–object duality (Shvarts et al., 2024).

Nevertheless, cognitive barriers persist, particularly with complex algebraic problems (Adu-Gyamfi et al., 2025), indicating that integrating relational reasoning into STEM learning must consider appropriate cognitive and instructional conditions (A. S. Liu & Schunn, 2017). Achieving a balance between procedural fluency and relational understanding is crucial for students to transition from rote memorization to flexible, meaningful, and contextual application, especially in Pythagorean Theorem learning.

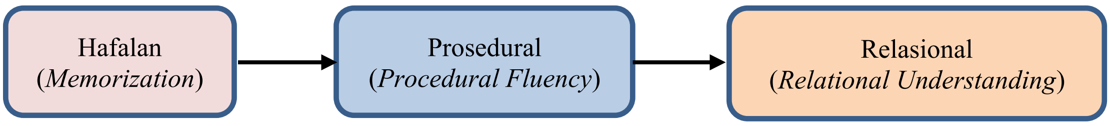


Figure 3. *Transition of Understanding: Memorization → Procedural → Relational*

Didactic Barriers and the Role of Instructional Design

Didactic barriers primarily result from the dominance of procedural approaches that neglect meaningful context, where teachers emphasize formula memorization without exploration opportunities (Kinnebrew et al., 2017). Research shows that Problem-Based Learning (PBL) can enhance cognitive engagement and reduce didactic barriers. Integrating cultural context (Ngu et al., 2018) and ethnomathematics-based instruction also broadens conceptual understanding (Payadnya et al., 2024). Thus, instructional design in Indonesia should adopt adaptive strategies, formative assessment, and technology integration to address didactic barriers more effectively.

Co-occurrence Patterns and Barrier Cycle Model

Co-occurrence analysis indicates strong associations between ONT–EPI (0.82) and EPI–DID (0.74), suggesting that prerequisite weaknesses (ONT) tend to trigger procedural errors (EPI), while unsupportive instructional strategies (DID) reinforce these errors. This pattern aligns with the vicious cycle concept in mathematics learning difficulties (Hiebert, 2007), in which unaddressed initial barriers recur in subsequent topics.

These findings confirm that student errors are systematic rather than random (Adorni et al., 2019; R. Liu et al., 2016). Pedagogically, appropriate instructional strategies are essential to break the barrier cycle (Fan et al., 2023; Lee & Bosch, 2024), and Research highlights the long-term impact of initial barriers on cognitive difficulties (Mareva & Holmes, 2021; Starling-Alves et al., 2025). Effective interventions require composite instructional designs (Loibl et al., 2024) and the implementation of a favorable error climate, allowing students to respond adaptively to failure (Dresel et al., 2025).

Local and Global Context Comparisons

Cross-country analysis shows significant differences in student failure rates on Pythagorean story problems: 80–100% in Indonesia, higher than 55–65% in Malaysia, and 40–60% in Australia, influenced by curriculum, assessment, and learning culture (Ngu et al., 2018). High barriers in Indonesia align with studies highlighting weak integration of conceptual understanding in the curriculum (Salas et al., 2024), although culturally and ethnomathematics-based approaches are beginning to show positive effects (Payadnya et al., 2024).

When examined alongside global literature (Albarracín et al., 2021; Gilligan-Lee et al., 2022), local context enriches understanding of socio-cultural factors influencing mathematics learning barriers. Cross-country differences appear driven by variations in curriculum sequencing, frequency of diagnostic assessments, and exposure to context-based learning.

Relevance and Limitations in the Indonesian Context

A key finding is the high prevalence of learning barriers in Indonesia compared to Malaysia and Australia, highlighting the strong influence of curriculum, assessment, and learning culture in addition to cognitive and epistemological factors. The Indonesian curriculum emphasizes mechanistic procedures with limited context, while assessments focus on final answers rather than thinking processes. These factors contribute to high failure rates in story problems and geometric visualization tasks.

This context indicates that, although the ONT–EPI–DID model is universal, its implementation must account for local factors. In Indonesia, instructional reform should focus on integrating contextualized learning, prerequisite diagnostic assessments, and teacher training to build visual and conceptual scaffolding. Cross-country comparisons show that success in reducing learning barriers is closely linked to how curricula and assessments support conceptual understanding, rather than mere procedural skills.

Theoretical Implications

The ONT–EPI–DID Model extends the frameworks of Brousseau (1997) and Duval (2017) with three main contributions: demonstrating dynamic interactions among barriers, integrating cross-national quantitative and qualitative evidence, and formulating a recurrent barrier cycle. The Model aligns with relational reasoning (Ferrara & Pozio, 2023) and strengthens the literature on epistemological barriers (Coutat et al., 2016; Dwyer, 2023), significantly contributing to international mathematics learning difficulties Research.

Practical Implications

Practically, the study underscores the importance of prerequisite diagnostic assessments to detect ontogenetic barriers early (Pettersen & Nortvedt, 2018), alongside real-world problem-based learning combined with visual-conceptual scaffolding to simultaneously reduce epistemological and didactic barriers (Coutat et al., 2016; van Nooijen et al., 2024). Process-oriented assessments emphasizing conceptual reasoning and reflection, rather than final answers alone, should be prioritized in line with policies supporting teacher training and context-based resource development.

Adoption of technological innovations, such as adaptive learning based on reinforcement learning (Gershman & Daw, 2017), intelligent tutoring systems (Bhalwankar & Treur, 2021), and adaptive network modeling (Gao et al., 2023), can provide personalized support and real-time feedback. However, implementation must consider digital infrastructure limitations in developing countries to avoid creating new inequities (Rasimin et al., 2024). Integrating pedagogical strategies and technology can break the ONT–EPI–DID cycle, enabling deeper, adaptive, and sustainable understanding of the Pythagorean Theorem.

4. SIMPULAN

This systematic literature review analyzed learning barriers in the Pythagorean Theorem at the junior high school level by integrating quantitative and qualitative data from 20 studies per the PRISMA 2020 protocol. The findings indicate that barriers do not occur in isolation but interact cyclically in the form of ontogenetic (ONT), epistemological (EPI), and didactic (DID) barriers, with prevalences of 85%, 75%, and 70%, respectively.

The main theoretical contribution is the development of the ONT–EPI–DID Barrier Cycle Model, which extends the frameworks of Brousseau (1997) and Duval (2017) through co-occurrence mapping and explains the mechanisms of the vicious cycle that reinforces mathematics learning difficulties, particularly in basic geometry. This study recommends prerequisite diagnostic assessments, real-world problem-based learning with visual–conceptual scaffolding, and process-oriented assessments to strengthen conceptual understanding. These recommendations are relevant in Indonesia and can be adapted to other educational contexts.

This study has several limitations that warrant consideration. First, despite employing a rigorous systematic literature review protocol, there is a potential publication bias as most included articles (60%) originate from Indonesia, which may limit cross-national generalizability. This dominance could reduce the representativeness of international findings, particularly given variations in curriculum, assessment practices, and learning cultures across countries. Second, the analyzed data are entirely secondary, meaning the findings reveal associative patterns rather than causal relationships among barriers.

In this regard, future Research should focus on empirically testing the ONT–EPI–DID Model in real classroom settings. One relevant approach is experimental Research, such as comparing the effectiveness of contextualized learning strategies and visual scaffolding in reducing students' epistemological barriers. An alternative approach is Design-Based Research (DBR), in which the Model can be examined through iterative cycles of design, implementation, evaluation, and refinement within Pythagorean Theorem instruction. Design-Based Research allows for developing adaptive and contextualized interventions while providing theoretical contributions toward model validation. Additionally, future studies could broaden the context by involving other countries, different educational levels, and affective–social factors that may reinforce or disrupt the barrier cycle.

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