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MISCONCEPTIONS IN LEARNING GROUP THEORY: A CASE STUDY OF PRE-SERVICE MATHEMATICS TEACHERS

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ABSTRACT

Penelitian ini bertujuan untuk mengeksplorasi miskonsepsi-miskonsepsi di perkuliahan Teori Grup pada mahasiswa calon guru. Penelitian ini merupakan bagian dari penelitian panjang disertasi tentang "abstraksi pada perkuliahan teori grup: studi fenomenologi hermeneutik pada mahasiswa calon guru". Metode penelitian ini menggunakan penelitian kualitatif dengan pendekatan studi kasus. Data yang diperoleh berupa *Focus Group Discussions* (FGD) dan catatan harian perkuliahan. Temuan dalam penelitian ini ada dua miskonsepsi yang terjadi yaitu dari *misundestanding* dan mispersepsi. Miskonsensi diantaranya tentang miskonsepsi sebuah grup sama dengan sebuah himpunan, sebuah grup bagian sama dengan sebuah himpunan bagian dan elemen invers dalam grup hanya sebagai pembagian dalam perkalian. Terdapat 2 miskonsepsi yang terjadi yaitu misinterpretasi dan *misunderstanding.*

This study aims to explore the misconceptions that occur in group theory learning conducted by pre-service mathematic teachers. This research is part of a long research from abstraction in learning group theory: study of hermeneutic phenomenology on pre-service mathematics teachers. This research method uses case study in qualitative research. Data obtained in the form of Focus Group Discussion (FGD) videos and pdf files of participant diaries. The findings of this study indicate a misconception on group as set and subset as subgroup and on the inverse element as only one over in multiplication. These two misconceptions show from misunderstanding and misperceptions.

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1. INTRODUCTION

Pre-services mathematics teachers must have knowledge of mathematics, knowledge of student learning, and knowledge of pedagogy (Harel, 2008). Harel (2008) said view of mathematical knowledge is divided into two sets of knowledge namely subject matter and conceptual tools. The subject matter is a collection of definitions, theorems, proofs, problems, and solving problems recognized by the mathematical community. While the conceptual tool is the process of thinking when you want to get the subject matter such as problem solving, reasoning, abstraction, and so on(Harel, 2008). The view is in line with Fischbein (1987) view of mathematical knowledge as two different sides. Based on (Fischbein, 1987) knowledge of mathematics can be seen from two sides namely knowledge of mathematics as formal knowledge and knowledge of mathematics as human activity. Formal knowledge is knowledge with institutional dimensions that are free from subject context and stored in treatises and recognized by the community. Knowledge of mathematics as human activity is invented by human beings. The procces of creating mathematics were like conceptual tools.

Group theory as a subject matter or formal component in learning mathematics that can develop higher-order thinking (Tim Kurikulum Prodi S1 IndoMS dan MiPAnet, 2013). In many colleges, group theory in the first course for students in which they must go beyond learning “imitative behavior patterns” for mimicking the solution of a large number variations on a small number problems (Dubinsky, Dautermann, Leron, & Zazkis, 1994). Group theory learning is a material that has a dominant formal aspect (Subroto & Suryadi, 2018; Weber & Larsen, 1998). The formal aspect of group theory can be seen from the knowledge structure in which there are definitions, theorems, proof of theorems, problems, and solving problems. Group theory is a topic in abstract algebra that talks about groups and their properties (Galarza, 2017). The group itself is a non-empty set with binary operations whose elements meet associative properties have identities and have inverses (Durbin, 2009; Gallian, 2017; Hungerford, 1974).

According (Cuoco, 2001) Concepts are perceived as knowledge buildings so misconceptions are errors in building knowledge. This error can occur because the knowledge foundation is not strong enough. misconceptions is a kind of misunderstandings and misinterpretations which is derived from inaccurate meanings (Ojose, 2015).

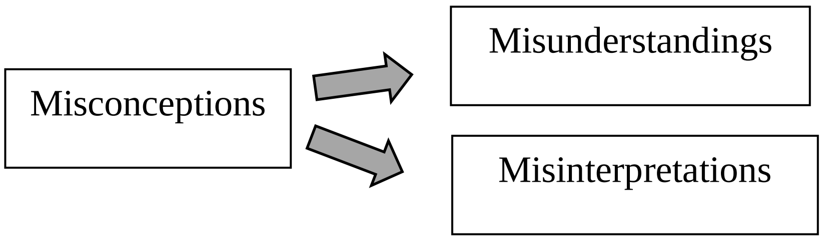

Figure 1. Kind of Misconceptions

Examples of misunderstandings that occur such as the multiplication operation will always increase. This examples true if only for positive numbers and integers then the case for negative numbers and multiplication of fraction less than one will be different. Examples of misinterpretations such as ¼ is greater than ½ because 4 is greater than 2. Misconceptions occur as a result of a lack of understanding of a concept (Albah, Suradi, & Minggi, 2015). Misconceptions arise due to the wrong choice of concept in understanding something or a decrease in understanding of a concept. Misconceptions are part of errors, in other words it can be stated that all misconceptions are errors, but errors are not necessarily misconceptions (Ay, 2017). This misconception occurs when they have a misconception one of which can be caused by internal factors (Wati & Saragih, 2018).

Misconceptions in group theory learning are closely related to previous knowledge (Ay, 2017). This occurs when understanding group theory is linked to student learning experiences. In addition to being related to previous knowledge, misconceptions occur because of making inaccurate conclusions about new knowledge. This misconception is important to reveal to see the actual cognitive reality of students. Clear cognitive reality can help model learning and emphasize the right group theory material.

2. METHOD

This research uses case study in qualitative research. case study method is an approach to research that facilitates exploration of a phenomenon within its context using a variety of data sources (Baxter & Jack, 2015; Yin, 2016). This research uncovers the misconceptions that were made by pre-service mathematic teachers when group theory learning.

2.1. Research Subject

The purpose of giving the proof problem is to see the thinking activities that the participants are doing. Participants were seen and observed from focus group discussion activity data and diaries. Subject Matter in Group Theory Learning includes group, subgroup, Isomorphisms, Homomorphisms, Cosets, and Quotient Groups. When the data needs to be deepened interviews are conducted to confirm the arguments of their answers.

2.2. Data Collecting

The data collected were in the form of Focus Group Discussion (FGD) activities and participant diaries. In FGD any participants can discussion about content in very deeply. In FGD, if discussion can label Participant 1 (P1), Participant 2(P2), P3, P4, P5, P6, and if researchers participating in the discussion are labeled with P0. All data collected is in Indonesia langunge so that the term “Partisipan” refer to Participant. This study focuses on looking at the thinking activities of 6 participants. The data is reduced and processed using the help of NVivo 12 which focuses on misconceptions.

2.3. Data Analysis

All collected data were analyzed based on the misconception theory presented in the introduction. Data in the form of videos is watched repeatedly to make sure there are misconceptions or not. The researcher also looked at the lecture diaries written by the participants. Diary reading is also done repeatedly to confirm whether there are misconceptions or not.

3. FINDINGS AND DISCUSSION

3.1. Findings

In this study several misconceptions about group theory were found. The following are the misconceptions that arise in this research:

a) Misconception Group as Set and Subgroup as Subset

One of these misconceptions occurs when participants are given a problem to prove the theorem about subgroups. The following is the discussion data from the FGD related to misconceptions:

Table 1. Conversation Partisipants

Timespan	Content
40:44,0 - 41:50,0	P1: *I'm confused what's the difference between subset and subgroup?*
	P6: *What is the difference between subgroup and subset sir?*
	P0: *Right if the subset is only a set while the subgroup is a set and operations*
	P1: *oo... just found out I...*

Findings of this data take place when participants are given the task of proving the theorem about subgroups *i.e.* "*Suppose* G *is a subgroup* $H \subseteq G$ *and* $H \neq \emptyset$. H *is said to be a subgroup of* G *if only if* $ab^{-} \in H$ *with* $b \in H$". When Participant 1 and Participant 6 discussed to prove the theorem. The data obtained from 6 participants showed the same trend in each participant it can be seen from the data obtained:

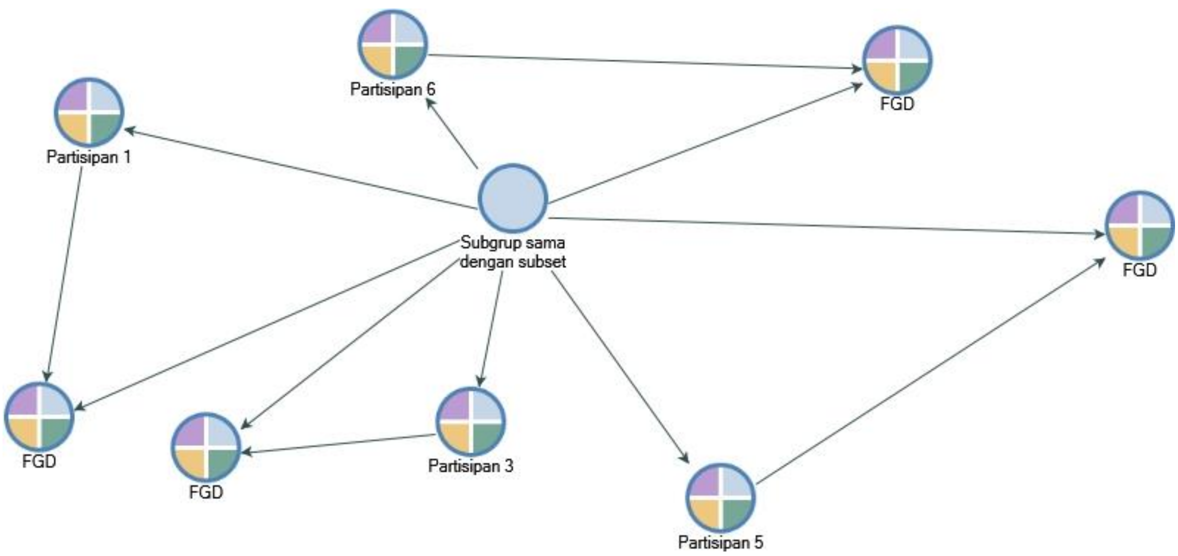

Figure 2. Data View from NVivo 12 about Group as Set and Subgroup as Subset

Based on Figure 2 In english "subgrup sama dengan subset" equal "subgroup as subset". The Figure 2 connections connected by arrows are the same coding "subgrup sama dengan subset" that appear in the data in the FGD and it can be seen that several participants experienced this misconception. The misconception data about this is obtained from several participants. This misconception occurs because of a misunderstanding of the group symbol which only shows the set. Even though the symbol is for ease of writing only, with the addition of the word group it definitely has a binary operation. The findings of this data are also the same in research conducted on this not only the first time. This misconception occurs when they have a misconception, one of which can be caused by internal factors (Wati & Saragih, 2018).

b) Misconception Invers Element as only One Over in multiplication ($\frac{1}{a}$)

When participants are given a problem about the inverse element, participants carry out solving activities using symbols. Participants understand the inverse whose symbol is $\frac{1}{a}$. Participants believed that the symbol a^{-1} always connotes $\frac{1}{a}$. This is due to his past learning experience that if a number is to the negative power of 1 then it means one per, whereas in this group theory the meaning of the inverse is broader and more general. The meaning of this inverse depends on the operations presented in the group, so it can take various forms. The following data were obtained about the meaning of the inverse:

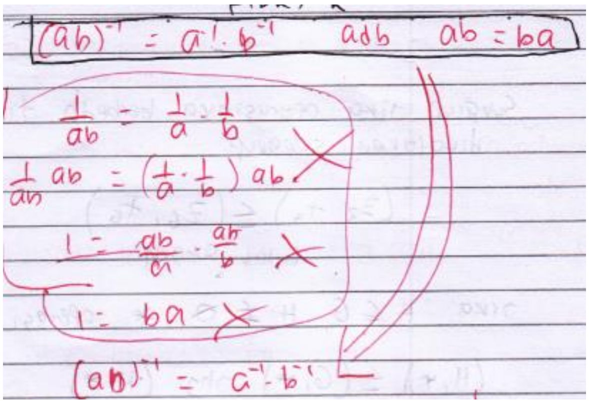

Figure 3. Activity Participant 5 About Solving Problem of Invers Element

Based on Figure 2. Participant 5 believes that to lead the proof, the goal is to bring up the inverse element of the ab element, namely $\frac{1}{ab}$. Understanding Participant 5 believes that because the operation is multiplication, the inverse element is one per ($\frac{1}{ab}$), even though from the beginning the researcher has made an agreement that the multiplication operation is only to facilitate understanding of operations in general. This has an impact on the meaning of the inverse itself on the inverse element of the multiplication operation, namely division.

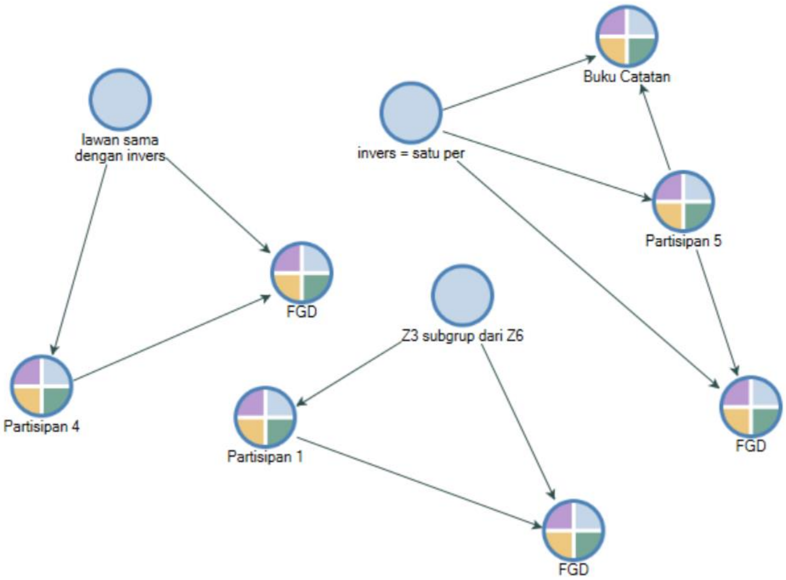

Figure 4. Misconception from Data View in Learning Group Theory

In Figure 3. The coding that "lawan sama dengan invers" is the same as inverse is versus as inverse and " invers = satu per" is inverse = one over in multiplication. The emergence of misconceptions apart from the FGD data can also be seen in the participant diary (P5). Based on the data obtained, the meaning of the inverse varies in the minds of the participants. This misconception shows the strongest misperception. The inverse element that is understood is only the inverse of multiplication. This misconception also arises from the understanding of the inverse symbol which is the same as the inverse symbol in multiplication. This happens from the use of simple language from the inverse meaning, such as the inverse of the existing operation and what is presented is the multiplication operation so that the inverse operation is division.

3.2. Discussion

Following is the synthesis and analysis of the data found with the misconception theory presented in the introduction. In the misconception of group as set and subgroup as subset prospective pre-service mathematic teachers experience problems seeing the definition of group as incomplete. Strong set learning experience becomes an obstacle to inserting the concept of binary operations as a pair of the set to understand the definition of the group as a whole. This is reinforced by the use of symbols in group definitions which often only use the set symbol. The findings of this misconception of group as subset and subgroup as subset have been found by Titova (1991). Misconceptions are related to wrong intuition in understanding a concept. This intuition is one component of human activity initiated by Fischbein (1987). As a result this misconception is included in the category of misinterpretation because seeing a concept is only partially and directly related to previous learning experiences. The same goes for misconceptions Z_3 subgroup from Z_6 regardless of the binary operation of the two sets are different.

In the inverse element as only one over in multiplication misconception occurs due to understanding the concept only with experience. The binary operating experience that is commonly used for these symbols is multiplication so that when understanding the inverse it will be connected directly to the multiplication inverse (one over in multiplication. Based on the research findings it can be seen that the misconceptions that occur in student teacher candidates when studying group theory are as follows :

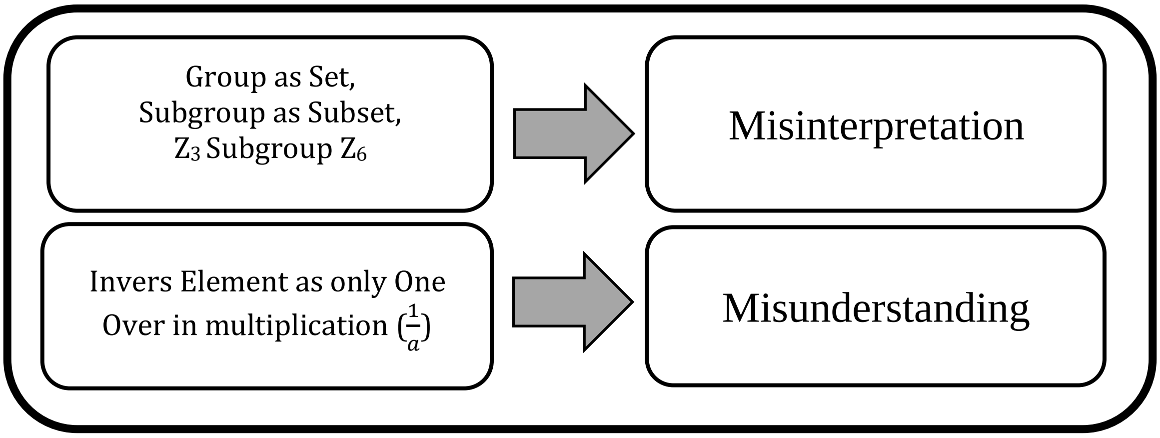

Figure 5. Misconceptions in Theory Group learning

Based on Figure 5 it can be seen that misconceptions in group theory lectures occur. Misconceptions that arise can be in the form of misinterpretation or misunderstanding.

4. CONCLUSION

Misconceptions that occur when learning group theory can occur. Misconceptions that arise can be in the form of misinterpretation and misunderstanding. Misinterpretation arises due to incomplete understanding of a concept. Misunderstanding arises as a result of being too close to a general concept with realistic knowledge that the learner has experienced.

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