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LEARNING DESIGN OF EXPONENTIAL NUMBERS THROUGH THE DESIGN OF A HYPOTHETICAL LEARNING TRAJECTORY USING DISCOVERY LEARNING

Dedi Muhtadi[1], Eko Yulianto[2], Redi Hermanto[3], Tiana Virawanti[4]

[1,2,3,4]Universitas Siliwangi, Jln. Siliwangi No. 24, Tasikmalaya 46115, Jawa Barat, Indonesia
E-mail: dedimuhtadi@unsil.ac.id

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ABSTRACT

Penelitian ini bertujuan untuk mengembangkan desain pembelajaran yang efektif dalam memahami konsep bilangan berpangkat dengan menerapkan model *Discovery Learning* menggunakan *Hypothetical Learning Trajectory* (HLT) sebagai landasan kerangka. Metode penelitian yang digunakan adalah metode *design research* yang terdiri dari tiga tahapan utama: *preliminary design*, *teaching experiment*, dan analisis *retrospektif*. Tahap awal melibatkan analisis literatur serta perancangan HLT yang disesuaikan dengan konsep bilangan berpangkat. Rancangan pembelajaran didasarkan pada HLT yang telah disiapkan dan diujicobakan melalui dua tahap eksperimen: *pilot experiment* dan *teaching experiment*. Analisis retrospektif dilakukan untuk membandingkan HLT awal dengan *Actual Learning Trajectory* (ALT) yang dicapai selama proses pembelajaran. Data dikumpulkan melalui observasi untuk memonitor aktivitas pembelajaran dan kondisi lingkungan serta melalui rekaman kegiatan pembelajaran untuk mendapatkan pemahaman tentang partisipasi siswa dalam eksperimen perancangan. Selain itu, Lembar Kerja Siswa (LKS) digunakan sebagai alat bantu bagi siswa dalam memahami konsep bilangan berpangkat, dan soal evaluasi digunakan untuk mengukur capaian hasil belajar siswa terhadap materi tersebut. Wawancara juga dilakukan untuk mendapatkan pemahaman yang lebih mendalam dari perspektif siswa. Hasil penelitian ini memberikan kontribusi signifikan dalam pengembangan metode pembelajaran matematika yang lebih efektif, khususnya dalam pemahaman konsep bilangan berpangkat, serta memberikan panduan bagi guru untuk merancang pembelajaran yang lebih relevan dengan kehidupan sehari-hari siswa. Implikasi hasil penelitian ini memberikan pedoman bagi guru dalam merancang pembelajaran yang lebih relevan dengan kehidupan sehari-hari siswa.

This research aims to develop an effective learning design for understanding the concept of exponentiation by applying the Discovery Learning model using the Hypothetical Learning Trajectory (HLT) as the framework foundation. The research method used is a design research method consisting of three main stages: preliminary design, teaching experiment, and retrospective analysis. The initial stage involves literature analysis and the design of an HLT tailored to the concept of exponentiation. The instructional design is based on the prepared HLT and tested through two experimental stages: pilot experiment and teaching experiment. Retrospective analysis is conducted to compare the initial HLT with the Actual Learning Trajectory (ALT) achieved during the learning process. Data is collected through observation to monitor learning activities and environmental conditions, as well as through recordings of learning activities to gain an understanding of student participation in the design experiments. Additionally, Student Worksheets (SW) are used as aids for students to understand the concept of exponentiation, and evaluation questions are used to measure students' learning outcomes in the subject matter. Interviews are also conducted to gain a deeper understanding from the student's perspective. The results of this research make a significant contribution to the development of more effective mathematics learning methods, particularly in understanding the concept of exponentiation, and provide guidance for teachers to design learning that is more relevant to students' daily lives. The implications of the research results provide guidelines for teachers in designing learning that is more relevant to students' daily lives.

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1. INTRODUCTION

In the discipline of mathematics, a strong understanding of exponents is a key element in forming a broader mathematical foundation. Exponents are a core concept that influences many more complex areas of mathematics, such as exponential functions, logarithms, and differential equations. Weber (2002) supports this view, stating that students' ability to master the concept of exponents is an important indicator of their ability to grasp advanced mathematics. Exponential functions are crucial in modeling growth and decay phenomena in various fields such as economics, biology, and physics (Caice, 2020).

Recent research underscores that understanding exponents is not only important in an academic context but also has significant relevance in everyday life. For instance, this concept is involved in calculating interest on loans, analyzing population growth, and scientific processes like radioactive decay (Weber, 2002). Arisandy and Fuad (2019) highlight the importance of understanding exponents in personal finance management, investment, and financial planning. However, findings from Nurkamilah and Afriansyah (2021) indicate that students face difficulties in understanding the concept of exponents, including challenges in identifying repeated multiplication that can be transformed into exponents and in illustrating real-life applications of the concept. Meldawati and Kartini (2021) also found several errors made by students in understanding exponents, including conceptual mistakes in using terms, concepts, rules, and formulas related to exponents. Therefore, a deep and accurate understanding of exponents is crucial for success in various aspects of life, both academically and practically.

Various studies have been conducted to address students' learning difficulties in exponents. Simanjuntak and Sihombing (2022) developed a learning design in the form of a mathematics module with a problem-solving strategy for exponents and roots, motivated by the low learning outcomes of students in this area. According to Wijaya (Cahirati, 2020), knowledge becomes meaningful for students when the learning process is conducted in a context. Zulkardi (Adha, 2019) also emphasizes the importance of context in mathematics learning, as it can help students understand abstract mathematical concepts. Research by Susanti et al. (2018) supports this by using the context of cell division in human body development in the teaching of exponents. The results showed that using context in teaching exponents can enhance students' understanding of the concept and improve their learning motivation.

This study uses the context of Amoeba reproduction as an initial approach to teaching exponents. Amoeba reproduction is a vegetative reproduction process that allows for the exploration of growth concepts. This approach aligns with the theory proposed by Van de Walle (Susanti, 2018), which emphasizes that the concept of exponents can be better understood through real contexts that link the meaning of exponents to concrete situations. Exponential functions naturally reflect various real-world problems involving growth or decay. The context of Amoeba reproduction is used as a starting point in teaching exponents, with the hope that students will find it easier to understand the concept and increase their enthusiasm for learning.

To address students' learning difficulties in understanding exponents, besides using relevant contexts, teachers also need to design learning activities that allow students to discover the concepts being taught. Piaget (Isrok'atun, 2018) posits that knowledge becomes more meaningful when students are actively involved in seeking and discovering it themselves. One instructional model that provides such opportunities is the discovery learning model, recommended in the 2013 Curriculum according to Permendikbud No. 103 of 2014. Eggen and Kauchak (2012) reveal that the proper implementation of the discovery learning model can lead to deep conceptual understanding in students, enhance long-term retention, and promote critical thinking skills.

The discovery learning model has several advantages relevant to teaching exponents. First, this model facilitates active student engagement in the learning process, allowing them to discover mathematical concepts and principles through exploration and experimentation, which can enhance their deep understanding. The application of the discovery learning model with a scientific approach has been found to improve mathematics learning by increasing student activity and engagement in the learning process (Okwina, 2020). Overall, research results show that the discovery learning model positively impacts students' cognitive engagement, which is essential for understanding complex concepts like exponents (Orr, 2016).

Second, by allowing students to discover on their own, they are better able to relate new concepts to their prior knowledge, thereby strengthening their understanding of exponents. Recent research by Rosmala et al. (2023) supports this, showing that learning using the discovery learning model encourages students to actively discover concepts and directly engage in organizing and connecting knowledge to conclude.

Additionally, the discovery learning model can increase students' interest in mathematics because they feel they play an active role in the learning process. Research by Rosmala et al. (2023) indicates that the discovery learning model stimulates students to actively explore mathematical concepts, fostering curiosity and engagement in the learning process. Students involved in discovery learning demonstrate higher motivation levels due to their active role in building their knowledge and understanding of mathematical concepts. This hands-on approach fosters curiosity and ownership of the learning process, leading to increased engagement and motivation (Khairunnisa et al., 2020).

Therefore, this model can create an interactive learning environment that facilitates the development of critical thinking skills and student creativity. Research by Khairunnisa et al. (2020) found that students engaged in discovery learning showed significant improvement in solving problems creatively and applying mathematical concepts in new situations.

Hence, this study aims to design a learning plan for exponents using the context of Amoeba reproduction through the discovery learning model. It is hoped that this learning design will help and facilitate students in a better understanding of the concept of exponents.

2. METHODS

This research adopts the design research method to develop a learning trajectory through the discovery learning model. Design research is a systematic approach to designing, developing, and evaluating educational programs, processes, and products (Akker et al., 2007). Gravemeijer & Cobb (2006) describe three steps in design research: preparing for the experiment, designing an experiment, and retrospective analysis.

The preparation phase begins with analyzing students' learning obstacles regarding the exponent's material, conducting exploratory interviews with teachers, studying the history of teaching the material, and researching related phenomena in the literature to generate ideas for creating relevant contextual problems. The next step is to design an initial Hypothetical Learning Trajectory (HLT) for teaching exponents, which includes learning objectives, learning activities, and learning hypotheses.

The design experiment phase involves implementing the learning process based on the initially developed HLT. This phase is carried out through a learning cycle consisting of a pilot experiment and a teaching experiment. The pilot experiment is an initial trial to gather data on adjustments and improvements needed for the HLT before it is implemented in a real classroom setting. Meanwhile, the teaching experiment is the stage for collecting data to answer research questions. The learning cycle occurring during the HLT implementation is illustrated in Figure 1 below.

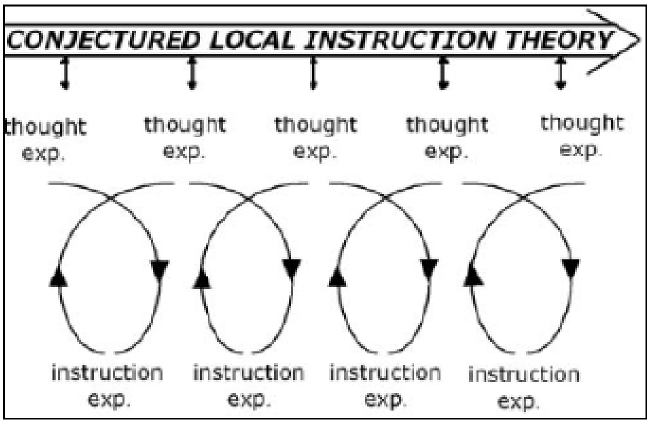


Figure 1. Design Research Cycle (Wijaya et al., 2021)

The final phase is the retrospective analysis, which involves comparing the initial HLT with the actual learning trajectory. The results of this analysis are used to refine the HLT for the next cycle.

2.1. Research Subjects

This study involved 60 ninth-grade students from a public junior high school in Tasikmalaya City for the 2023-2024 academic year. The research was conducted from August 5 to December 10, 2023. The research subjects were divided into two groups: 30 students from class IX A and 30 students from class IX B. Class IX B served as the pilot experiment group, while class IX A was the teaching experiment group. Both classes had similar academic performance and characteristics and had not received instruction on the topic of exponents according to information from their mathematics teacher.

2.2. Data Collection

The data in this study are qualitative and were collected through observations, interviews, and documentation. Observations were conducted during the implementation of the learning process, both in the pilot experiment and the teaching experiment, using observation sheets to record student activities, particularly their engagement in each learning phase. Interviews were used to confirm and clarify findings, especially those related to the learning trajectory that aligns with the set objectives. Meanwhile, documentation consisted of authentic data on student activities, including work results and learning videos. This documentation provided detailed insights into student activities during the learning process.

2.3. Data Analysis

Data analysis employed qualitative analysis techniques with descriptive, transcription, and clarification methods (Astuti & Wijaya, 2020). The descriptive method was used to describe the situations and conditions during the learning process. The transcription method was used to transcribe video and interview recordings into written form. The clarification method was used to interpret observations during the learning process. The initial HLT was compared with the data analysis results from the actual learning process to identify necessary changes based on students' evolving thinking. From this analysis, the HLT was adjusted based on new insights emerging from students' thinking.

3. RESULTS AND DISCUSSION

3.1 Results

The results of this research include a learning trajectory that helps students understand the concept of exponents through the context of amoeba reproduction. This study used an HLT tailored to the lesson plan as a guide for the learning process. The HLT serves as a conceptual map for students to follow during the lessons. The design of learning activities for exponents consists of four activities designed based on the HLT and the hypothesized outcomes of students' thought processes, using the context of amoeba reproduction, implemented through the discovery learning model. The objective is for students to determine the concept of exponents from repeated multiplication and to determine the results of exponentiation.

During the pilot experiment phase, the researcher tested the designed learning plan. Based on observations, students' worksheets (SW), test questions, and student interviews during the pilot experiment, it was concluded that student activities aligned with the targets set in the HLT design. However, some students experienced confusion and difficulty in completing tasks on the SW, requiring guidance and direction from the teacher. The HLT was revised by modifying several questions in the SW to ensure that all students could easily follow the planned learning trajectory. The revised HLT was then tested during the teaching experiment phase.

Meeting 1

The first meeting began with an introduction where the teacher reviewed prerequisite material, specifically the multiplication of integers. The teacher provided several example problems related to the multiplication of positive and negative integers. Mastery of this prerequisite material is crucial as it facilitates students' understanding of exponents.

After that, the teacher asked students to open the SW provided to each group. Students were instructed to work on the SW. The details of observations, analysis, and evaluation of the pilot experiment are as follows:

The first activity involved illustrating amoeba reproduction as a form of repetition or repeated multiplication. This served as the introduction to the lesson, where students were asked to recall the amoeba reproduction process by watching a video depicting this process. The video was then used as a context to discover the concept of exponents. Next, students were asked to understand and formulate problems related to amoeba reproduction presented in the SW. The students' work results in this activity can be seen in Figure 1 below.

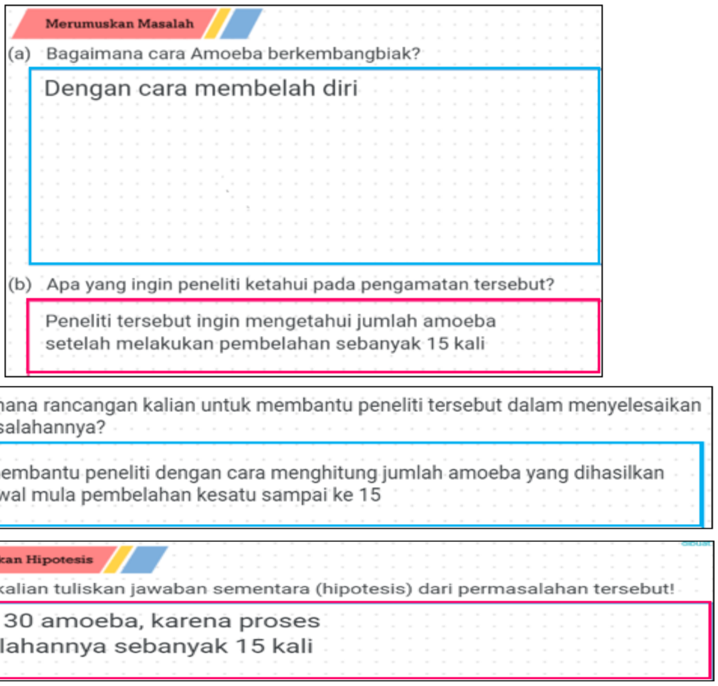


Figure 1. Students' SW Answers for the First Activity

From Figure 1, it is evident that students understand the amoeba reproduction process. Following this, students were asked to illustrate the amoeba division process from the initial stage through the first, second, third, and fourth divisions. The results of the students' illustrations can be seen in Figure 2 below.

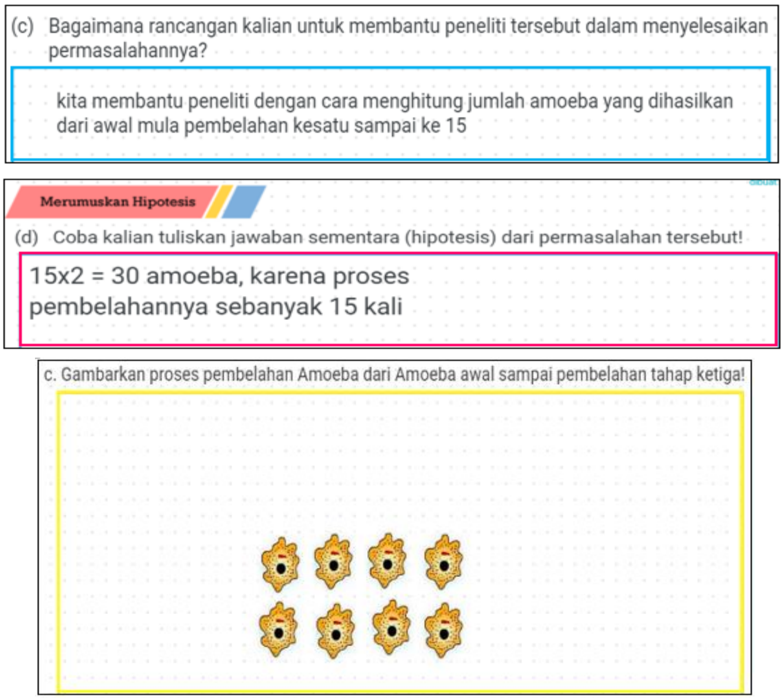


Figure 2. Students' SW Answers for the First Activity

In Figure 2, it is evident that students successfully illustrated the amoeba division process from the initial stage through the first, second, third, and fourth divisions as examples of duplication or repetition. The students' responses align with the predictions made by the researcher in the HLT.

The second activity involved determining the number of amoebas at each division stage. This activity aimed to demonstrate the process of duplication or repetition of a number that occurs during amoeba reproduction. Students were asked to calculate the number of new amoebas formed at each stage based on the illustrations they had made. Students were able to accurately determine the number of new amoebas emerging at each stage.

Next, the third activity involved determining the form of repeated multiplication using the same number to explain the number of amoebas at each division stage. The teacher emphasized the importance of understanding the meaning of the problems presented. In Figure 3, students are seen discussing with their group members to complete the tasks on the LKS in the third activity.

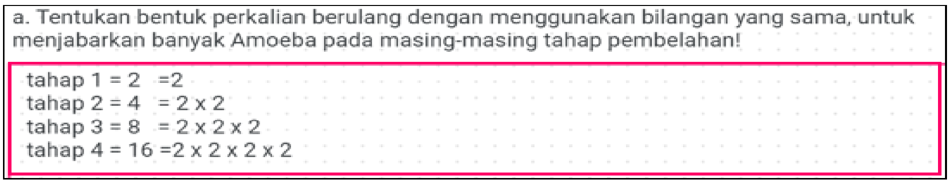
Figure 3. Students Discussing and Working on Tasks in the SW

During the discussion process, one group of students showed uncertainty in determining the form of repeated multiplication using the same number to explain the number of amoebas at each division stage. As a result, they sought help from the teacher. The following is the dialogue that took place:

Dialogue 1

Student 1	**:**	*Sir, how do we solve this problem? (showing the problem).*
Teacher	**:**	*Try to read and understand the instructions of the problem again.*
Student 2	**:**	*Oh, wait, Sir, repeated multiplication uses the same number at each stage, and there are four division stages, right? For instance, in the second stage, there are four amoebas produced, so the repeated multiplication form of four is 2×2, is that correct?*
Teacher	**:**	*Yes, that's correct.*
Student 2	**:**	*Sir, at the first stage, there are only two amoebas, so there's no repeated multiplication using the same number, right? How do we handle that?*
Teacher	**:**	*Is there a repeated multiplication form that results in two?*
Student 1	**:**	*2 x 1, Sir.*
Teacher	**:**	*Pay attention to the problem statement; it must use the same number. Are one and two the same number?*
Student 2		*No, Sir, but there isn't any other multiplication that results in two.*
Teacher		*If there isn't, then the repeated multiplication form for the first stage is simply two.*
Student 2		*Okay, Sir, so at the third stage, there are eight amoebas, so the repeated multiplication form is 2×2×2, right?*
Student 1		*Why not 4×2, Sir?*
Teacher		*We need to use the same number; if it's 4×2, those are two different numbers.*
Student 1		*Oh, I understand.*
Teacher		*Correct, the numbers used must be the same. Now, proceed to the next stage with the same rule.*

After discussions like the one in Dialogue 1, while working on the third task, students still felt uncertain about answering the given problem. However, after consulting with the teacher, the students managed to complete the task. Eventually, they were able to write the repeated multiplication form using the same number to explain the number of amoebas at each division stage. The students' work results can be seen in Figure 4 below.


a. Tentukan bentuk perkalian berulang dengan menggunakan bilangan yang sama, untuk menjabarkan banyak Amoeba pada masing-masing tahap pembelahan!

tahap 1 = 2 =2
tahap 2 = 4 = 2 x 2
tahap 3 = 8 = 2 x 2 x 2
tahap 4 = 16 =2 x 2 x 2 x 2

Figure 4. Students' SW Answers for the Third Activity

The fourth activity aimed to develop students' understanding of the concept of exponents through repeated multiplication and exponentiation. In this activity, students were given several tasks to complete. The first task involved filling out a table in the LKS, which aimed to help students analyze the concept of exponents. The teacher explained that there was a consistent pattern between the rows of the table, and students were asked to identify this pattern to facilitate filling it out.

During the discussion, students encountered difficulty filling in the "Repeated Multiplication Form" column for the 20th and 50th divisions, as the space provided was insufficient for writing out the complete answer. Through the teacher's guidance, students were introduced to a more simplified form of writing, allowing them to express repeated multiplication for large numbers. The results of the student's work on this task are depicted in Figure 5 below.

Berdasarkan jawaban yang kalian peroleh, lengkapi tabel di bawah ini!

dibuat: 30/07

Tabel Pembelahan Amoeba

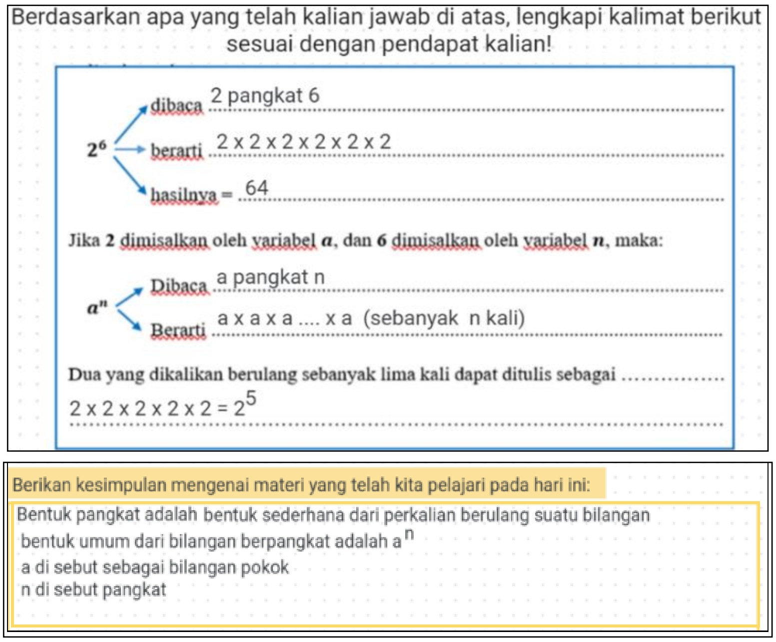
Pembelahan ke-	Bentuk Perkalian Berulang	Banyak Pengulangan Bilangan yang Sama	Bentuk Pangkat	Banyak Amoeba
1	2	1	2^1	2
2	2 x 2	2	2^2	4
3	2 x 2 x 2	3	2^3	8
4	2 x 2 x 2 x 2	4	2^4	16
5	2 x 2 x 2 x 2 x 2	5	2^5	32
⋮	⋮	⋮	⋮	⋮
10	2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2	10	2^{10}	1024
⋮	⋮	⋮	⋮	⋮
20	2x2x2x2x2...x2 sebanyak 20 kali	20	2^{20}	?
⋮	⋮	⋮	⋮	⋮
n	2x2x2...x2 sebanyak n kali	n	2^n	?

Keterangan:
Tanda tanya (?) tidak perlu diisi.

Figure 5. Students' SW Answers for the Fourth Activity

In Figure 5, students demonstrated a good understanding of determining exponentiation forms. However, one group still experienced confusion when filling in the row for the nth Division, as the division column was represented by the variable n. The teacher-guided by suggesting that students observe the pattern formed in the previous rows so that they could complete the row for the nth Division. Consequently, students completed the table accurately according to the instructions in the SW.

Next, the teacher instructed the students to draw conclusions based on the information they had found through several questions provided in the SW. The aim was to reinforce students' understanding of the concept of exponents. The results of the student's work are depicted in Figure 6 below.



Berdasarkan apa yang telah kalian jawab di atas, lengkapi kalimat berikut sesuai dengan pendapat kalian!

2^6 dibaca 2 pangkat 6

berarti 2 x 2 x 2 x 2 x 2 x 2

hasilnya = 64

Jika **2** dimisalkan oleh variabel a, dan **6** dimisalkan oleh variabel n, maka:

a^n Dibaca a pangkat n

Berarti a x a x a .... x a (sebanyak n kali)

Dua yang dikalikan berulang sebanyak lima kali dapat ditulis sebagai ....

$2 \times 2 \times 2 \times 2 \times 2 = 2^5$

Berikan kesimpulan mengenai materi yang telah kita pelajari pada hari ini:

Bentuk pangkat adalah bentuk sederhana dari perkalian berulang suatu bilangan

bentuk umum dari bilangan berpangkat adalah a^n

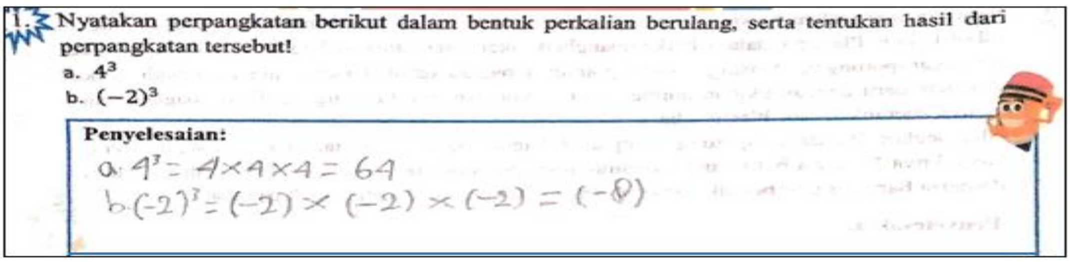
a di sebut sebagai bilangan pokok

n di sebut pangkat

Figure 6. Students' SW Answers During Conclusion

In Figure 6, it can be seen that students successfully understood the concept of exponents from the form of repeated multiplication and were able to determine the result of the exponentiation of a number. The teacher asked some students to present the conclusions they had drawn from the learning activities. The conclusions expressed by all student groups were consistent in meaning. Subsequently, the teacher provided additional explanations related to the concept of exponents as presented by several students.

Throughout the activities, it was observed that students were meticulous in solving the LKS problems, although they occasionally required guidance from the teacher to overcome certain questions. Through this guidance, students eventually grasped the concept of exponents. The teacher then assigned several practice problems related to the concept. The results of the students' work on these practice problems are shown in Figure 7 below.



1. Nyatakan perpangkatan berikut dalam bentuk perkalian berulang, serta tentukan hasil dari perpangkatan tersebut!

a. 4^3

b. $(-2)^3$

Penyelesaian:

a. $4^3 = 4 \times 4 \times 4 = 64$

b. $(-2)^3 = (-2) \times (-2) \times (-2) = (-8)$

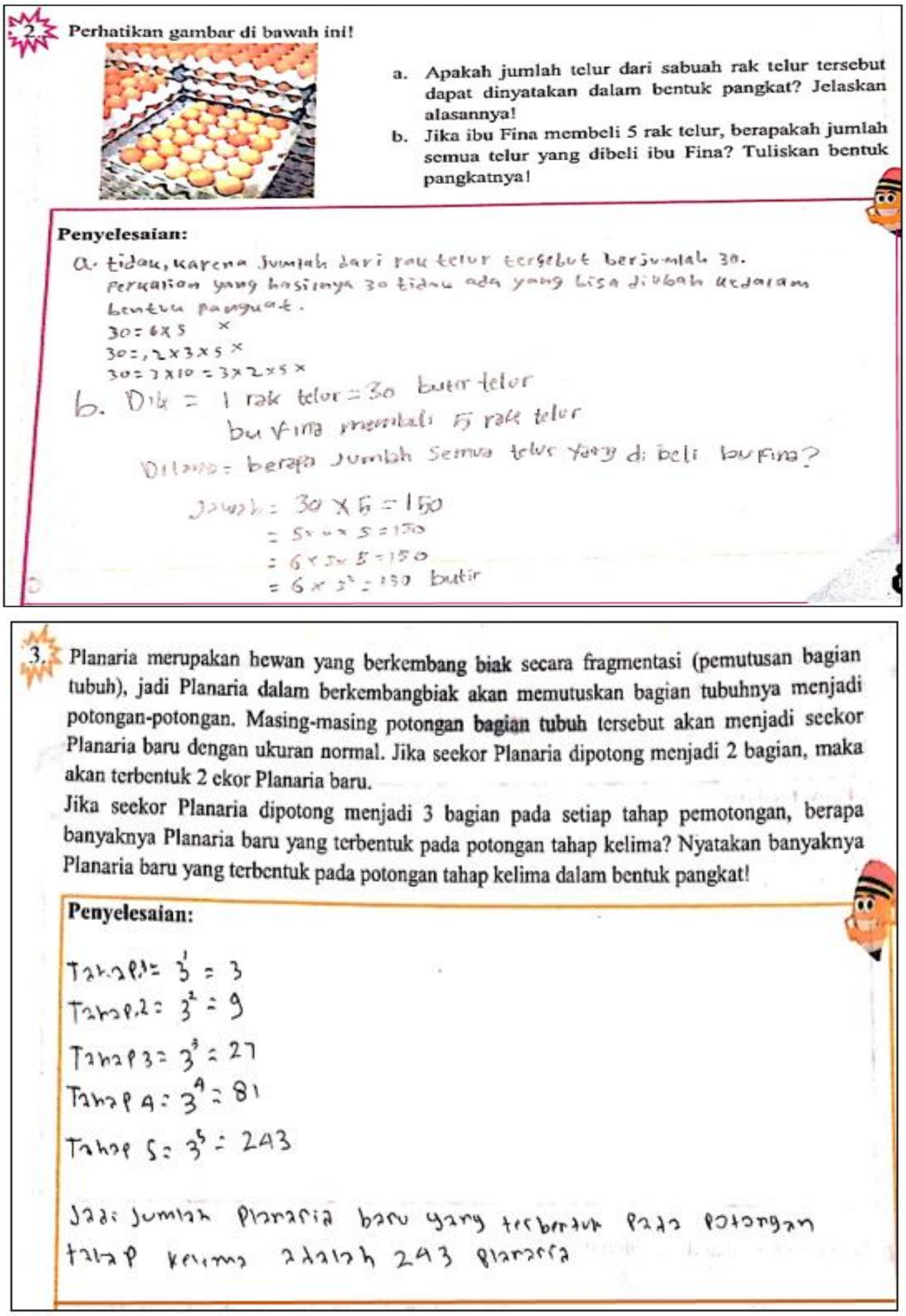


Figure 7. Students' Answers to Practice Problems

Based on the students' answers in Figure 7, it is evident that most students successfully solved several problems correctly. However, some students did not include conclusions in their answers.

Meeting 2

In the second meeting, the teacher administered a written test to evaluate the student's understanding of the concepts learned previously. This test was taken by 29 students from class IX-A. The results of the written test showed that the average student score was 94.35. The Minimum Competency Criterion (MCC) for mathematics in class IX at the school is 76. The scores from the written test for class IX-A are presented in Table 1 below.

Table 1. Written Test Score Categories

Category	Score	Frequency	Percentage
Fassed	≥ 76	28	96,55%
Not Passed	< 76	1	3,45%

From the implementation of the teaching experiment, it can be concluded that the series of activities designed in the HLT successfully helped students understand the concept of exponents.

3.2. Discussion

Students' understanding of exponents can be enhanced through a learning trajectory design that utilizes the context of amoeba reproduction. The learning activities allowed students to discover the concept of exponents through a series of actions, starting with the teacher showing a video of amoeba reproduction. This context was deemed suitable for the student's environment and aligned with the biology material they had previously studied. Choosing a context familiar to students is important, as Adha and Refianti (2019) state that a recognizable context helps students understand mathematical problems. Additionally, Surgandini et al. (2019) argue that using context can increase enjoyment in learning, motivate students to participate more actively, and reduce the perception that mathematics is abstract. Previous studies have also applied context in mathematics learning, such as using snakes and ladders games (Putri et al., 2020) and the context of human body development (Susanti et al., 2018) in learning exponents.

Amoeba reproduction serves as a relevant context for understanding exponents, helping students build a foundational understanding of exponential growth. The process of amoeba reproduction, a method of vegetative reproduction in organisms, provides a basis for understanding exponential growth. Van de Walle (Susanti, 2018) suggests that exponential concepts are evident in real-world situations, such as in organism growth. Exponential functions are often used to describe various real-world problems involving growth or decay.

The results of the teaching experiment show that in the initial activities, students could accurately depict the amoeba reproduction process, noting the doubling that occurs through diagrams or schemes they created. Students could then calculate the number of new amoebas at each stage, observing the pattern of doubling the previous number. This understanding led them to the concept of repeated multiplication, introducing them to exponents. Subsequently, students could explain the concept of exponents using their language and understanding.

The amoeba reproduction context proved useful in developing students' understanding of basic exponent concepts in grade IX. This context serves as a beneficial starting point in the learning process to facilitate students' comprehension of exponents. This aligns with Treffers & Goffree's view (Wijaya, 2012) that one important function of context is to form concepts, and thus, the context must include relevant mathematical concepts that allow students to construct them naturally.

Besides the activities undertaken by the students, the use of amoeba reproduction in forming basic exponent concepts is supported by the written test results. The evaluation showed that students could accurately solve problems related to exponents. The test results after applying the method indicated that the average student score was 94.35, surpassing the Minimum Competency Criterion (KKM) for grade IX mathematics, which is 76. The passing percentage was 96.55%, with 28 out of 29 students achieving the criterion.

Furthermore, this study provides a learning trajectory for exponent concepts through the discovery learning model using amoeba reproduction. It also reveals students' strategies in developing their understanding of exponents. These strategies result from implementing, testing, and analyzing the designed Hypothetical Learning Trajectory (HLT) in the pilot experiment, then revising it for the teaching experiment, which produced the Local Instructional Theory (LIT).

The retrospective analysis of the first cycle (pilot experiment) revealed several challenges students faced while following the designed learning trajectory. These challenges included difficulties in determining the number of amoebas at the 50th division stage due to large numbers requested in the table, challenges in completing the problem with the form due to the exponent being represented as a variable, and student responses not aligning with the predicted HLT. In response, the researcher revised the activities, including adjusting the anticipated student and teacher responses and revising the table and problems in the fourth activity. The revisions aimed to ease students' analysis of the given problems. Detailed revisions are found in Appendix Section 3 of the revised SW. These findings align with Mariam et al.'s (2017) study, which indicates that retrospective analysis results in a revised learning design based on prior implementation results.

The revised HLT was then tested in the second stage (teaching experiment) to address research questions. Changes made in the second cycle successfully addressed several difficulties encountered in the first cycle, indicating that all students could follow the designed learning trajectory. Based on the student's activities, it can be concluded that students successfully understood the basic concept of exponents through the learning trajectory designed with the amoeba reproduction context. According to Hadi (Prahmana, 2017), the entire process of developing and refining the instructional sequence results in Local Instructional Theory (LIT). Thus, the findings of this study are integral to developing LIT.

The resulting learning trajectory comprises a series of activities that meet the characteristics of the discovery learning model. According to Sanjaya (Isrok'atun, 2018), the first characteristic is emphasizing students' activities to search and discover. The learning process involved four activities. The first activity was that students could depict amoeba reproduction as doubling or repeating. This began with a video of amoeba reproduction to stimulate students' knowledge of the process. Students realized that amoeba reproduction could be applied in mathematics learning, aligning with Van De Walle's (Susanti, 2018) view that exponents reflect real-world problems involving growth or decay. Subsequently, students understood and formulated problems related to amoeba reproduction and illustrated the amoeba division process from the beginning to several subsequent stages, leading to understanding the doubling process through diagrams or schemes they created.

In the second activity, students determined the number of amoebas formed at each division stage. In the third activity, they identified repeated multiplication using the same number to explain the number of amoebas at each division stage, leading them to define the concept of exponents. This is supported by Arieyantini et al. (2017), who emphasize that the numbers must be the same for repeated multiplication. In the fourth activity, students established the exponent concept from repeated multiplication and determined the result of raising a number to a power. This began with completing the SW table related to amoeba reproduction to help students analyze and discover the exponent concept, concluding with their findings to reinforce their understanding.

The second characteristic, according to Sanjaya (Isrok'atun, 2018), is that all student activities are directed towards independently finding answers to the posed questions. The learning activities implemented problem-solving strategies to discover a concept from a given question. In this context, students solved problems related to amoeba reproduction to find the exponent concept. The learning process involved answering questions independently to discover the exponent concept, with the teacher acting as a facilitator, providing guidance and support to focus on the inquiry and discovery activities. This approach aligns with Piaget's theory (Isrok'atun, 2018), which posits that knowledge becomes more meaningful when students discover it themselves.

The third characteristic, according to Sanjaya (Isrok'atun, 2018), is developing critical, logical, and systematic thinking and intellectual development as part of mental processes. Discovery learning allows students to develop ideas or solutions to given problems, enhancing their thinking process in finding the concept being studied. In this study, the teacher played an active role in determining problems and their solution stages, while students faced relevant mathematical tasks to solve, discovering the exponent concept.

Based on activities meeting the discovery learning model characteristics, it was found that this approach substantially supports students in learning exponents. Discovery learning emphasizes active student roles in exploring and experimenting to construct their knowledge (Ozdem-Yilmaz & Bilican, 2020). Additionally, Alfieri et al. (2011) state that students involved in discovery learning tend to retain their understanding longer because they uncover principles and develop deeper comprehension of the material. Therefore, applying the discovery learning model using amoeba reproduction as an example can be considered a potential alternative in exponent learning.

4. CONCLUSION

The design of HLT integrating the amoeba reproduction context in exponent learning through the Discovery Learning model significantly enhances students' understanding of exponents. The HLT in this study is a learning trajectory designed based on initial learning trajectory hypotheses, including contextual problems, modeling, knowledge building, and formal mathematics. The final HLT obtained after implementation includes: (1) depicting amoeba reproduction as doubling or repeating, (2) determining the number of amoebas at each division stage, (3) identifying repeated multiplication using the same number to explain the number of amoebas at each division stage, and (4) establishing the exponent concept from repeated multiplication and determining the result of raising a number to a power. Recommendations from this study are: (1) research on HLT design contributes positively to students' understanding of exponents, suggesting similar research could design more comprehensive exponent learning HLT, and (2) designing learning sequences through presented problems is crucial and requires thorough examination so that students can follow them according to the intended learning goals.

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